# Predictions with Lattice QCD

## Fermilab Lattice, MILC, and HPQCD Collaborations

The central theme of *elementary particle physics* is to find new interactions of matter, energy, space, and time. When the matter in question is the quarks, one is faced with *quark confinement*: quarks never appear freely; they are always bound inside *hadrons*—baryons like the proton, or mesons like the pion or kaon.

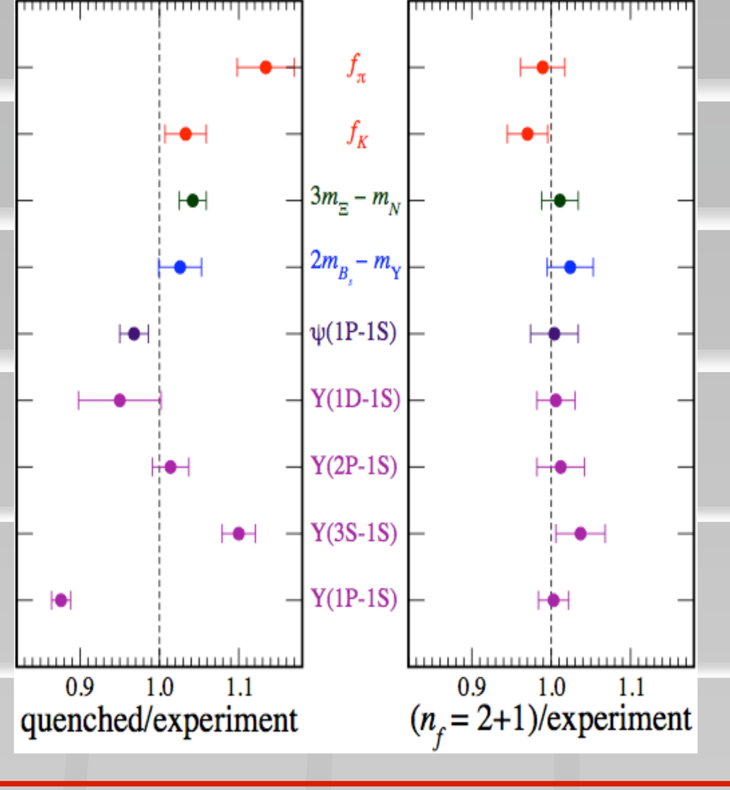
In the Standard Model of elementary particles, confinement is a phenomenon of quantum chromodynamics (QCD), the gauge theory of the strong force. Only hadrons can be detected, so to enable experiments to discover new interactions of quarks, the effect of confining the quarks into hadrons must be calculated with QCD. The best technique for doing the calculations is to formulate QCD on a space-time lattice. Lattice QCD and the Feynman path integral reduce the problem of to an integral whose dimension scales with the (linear) lattice size $N$ as $N^4$. We carry out the integration with Monte Carlo methods and importance sampling.

Any large-scale computing endeavor is a bit inscrutable, so it becomes more persuasive if the code's combination of principle and pragmatism can make predictions.

## *Background*: Postdictions Agree with Experiment

C.T.H. Davies, E. Follana, A. Gray, G.P. Lepage, Q. Mason, M. Nobes, J. Shigemitsu, H.D. Trottier, M. Wingate,
C. Aubin, C. Bernard, T. Burch, C. DeTar, Steven A. Gottlieb, E.B. Gregory, U.M. Heller, J.E. Hetrick, J. Osborn, R. Sugar, D. Toussaint,
M. DiPierro, A. El-Khadra, A. S. Kronfeld, P.B. Mackenzie, D. Menscher, J. Simone
(HPQCD, MILC, and Fermilab Lattice Collaborations)
*Phys. Rev. Lett.* **92**, 022001 (2004) [arXiv:hep-lat/0304004].

Some lattice QCD calculations omit sea quarks—called the quenched approximation. Results deviate from experiment. With 2+1 flavors of sea quarks (up, down, strange) we now find agreement for a wide range of masses, mass splittings, and matrix elements. Some of the ingredients used to obtain these results are not (yet) rigorously proven, leading to some controversy: the sea and valence quarks are not treated completely the same, leading to (suppressed) discretization errors that violate unitarity (*cf.* optical theorem).




## Test Ingredients via Predictions

Below we present several *predictions*: calculations that were finished before suitably precise experimental measurements were made. In this poster we emphasize the comparison with experiment and how the comparisons test the three key ingredients. We cover $D$-meson semileptonic form factors $f_+(q^2)$, $D$-meson decay constants $f_D$, and the mass of the $B_c$ meson. More ★s imply a more stringent test.

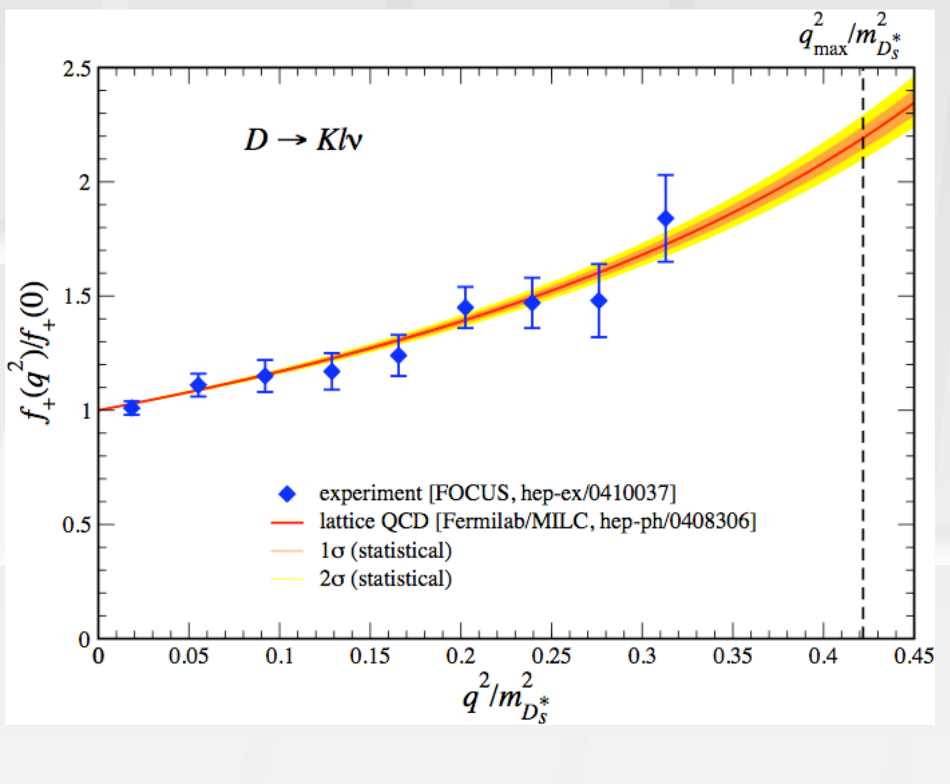
| *calculation* | *light sea* | *light valence* | *heavy discret.* |
|---|---|---|---|
| semileptonic $f_+(q^2)$ | ★★ | ★★ | ★ ★ |
| leptonic $f_D$ | ★★ | ★★★ | ★ ★ |
| $B_c$ mass | ★★ | — | ★★★ |

## Semileptonic $D$ Decays

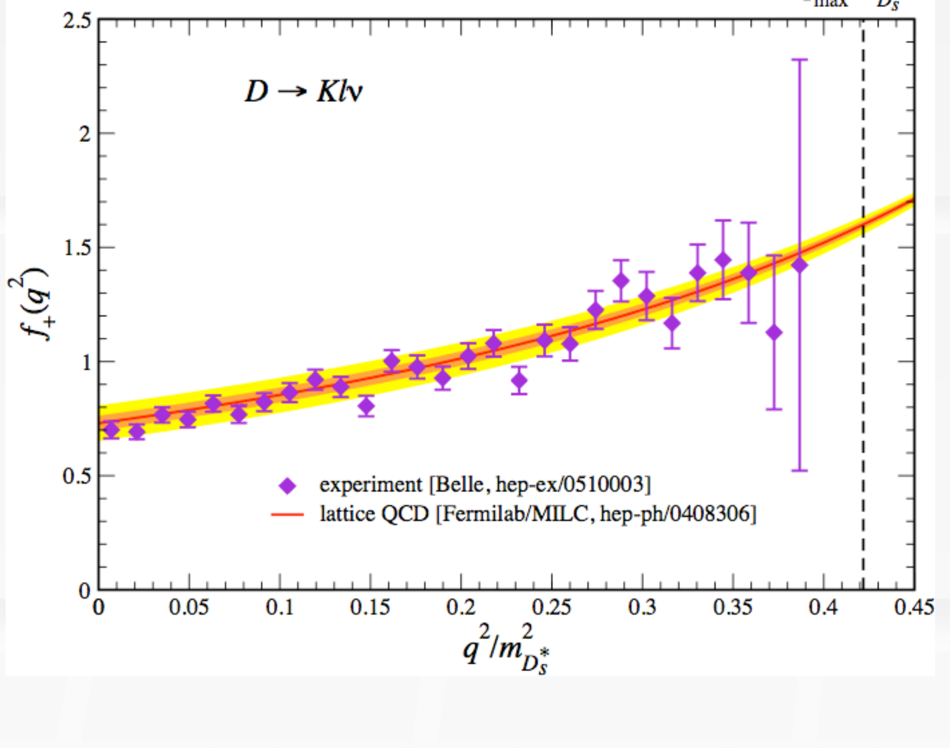
C. Aubin, C. Bernard, C. DeTar, M. DiPierro, A. El-Khadra, Steven Gottlieb,
E.B. Gregory, U.M. Heller, J. Hetrick, A.S. Kronfeld, P.B. Mackenzie,
D. Menscher, M. Nobes, M. Okamoto, M.B. Oktay, J. Osborn, J. Simone,
R. Sugar, D. Toussaint, H.D. Trottier
(Fermilab Lattice, MILC, and HPQCD Collaborations)
*Phys. Rev. Lett.* **94**, 011601 (2005) [arXiv:hep-ph/0408306].

When our paper was posted on the arXiv, we knew the normalization of the $D \to \pi l\nu$ and $D \to K l\nu$ form factors agreed with measurements from the BES Collaboration and the CLEO Collaboration. This agreement had been seen throughout the cycle of conference proceedings and journal publications. So this is almost, but not quite, a *prediction*.

More spectacularly, two months after our paper was posted on the arXiv, the FOCUS Collaboration finished a measurement of the shape of the $D \to K l\nu$ form factor. Their data are plotted over our curve (with 1 and $2\sigma_{stat}$ bands) and agree excellently.




In October 2005, the Belle Collaboration presented a more precise set of measurements, of both shape and normalization [hep-ex/0510003, hep-ex/0604049].




This success bodes well for lattice QCD calculations of the form factor for $B \to \pi l\nu$, which is a key part of the search for new $b$-quark decays.

## Leptonic $D$ Decays

C. Aubin, C. Bernard, C. DeTar, M. DiPierro, E. D. Freeland, Steven Gottlieb, U. M. Heller, J. E. Hetrick, A. X. El-Khadra, A. S. Kronfeld, L. Levkova, P. B. Mackenzie, D. Menscher, F. Maresca, M. Nobes, M. Okamoto, D. Renner, J. Simone, R. Sugar, D. Toussaint, H. D. Trottier
(Fermilab Lattice, MILC, and HPQCD Collaborations)
arXiv:hep-lat/0506030 → *Phys. Rev. Lett.*

QCD's influence on the leptonic decay $D_{(s)} \to l\nu$ is parameterized by decay constants $f_D$ and $f_{Ds}$. Until 2005, the only measurements were based on only a few events and had, hence, uncertainties of 20–60%.

For the 2005 Lepton-Photon Symposium, CLEO-c planned to announce a measurement of $f_D$ with 5–10% uncertainty. The challenge to (lattice) QCD was set.

We took up the challenge, finding [hep-lat/0506030]

$$f_{D^+} = 201 \pm 3 \pm 17 \text{ MeV},$$
$$f_{Ds} = 249 \pm 3 \pm 16 \text{ MeV}.$$

Afterwards, CLEO-c showed its new result. With 47 ± 8 events [hep-ex/0508057]

$$f_{D^+} = 223 \pm 17 \pm 3 \text{ MeV}$$

At this year's Moriond Winter Conference, the BaBar Collaboration showed a nice measurement of $f_{Ds}$ [http://moriond.in2p3.fr/EW/2006/Transparencies/J.W.Berryhill.pdf]:

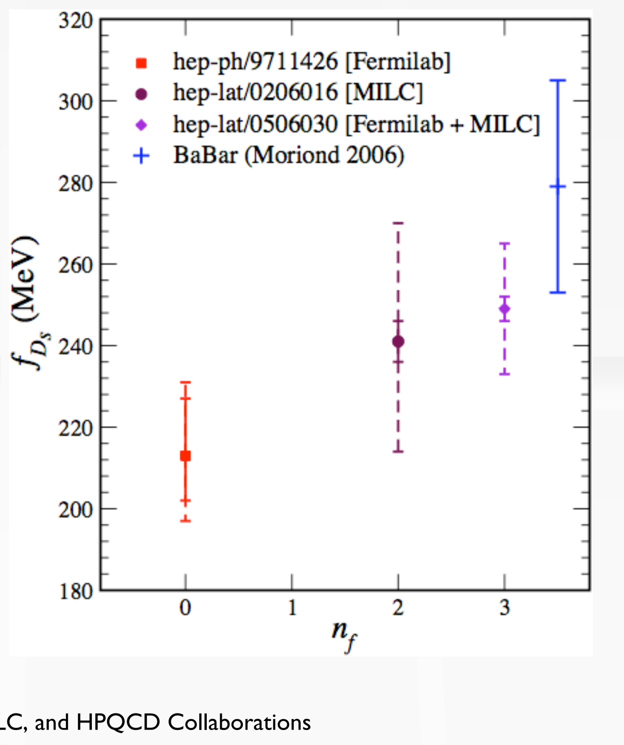
$$f_{Ds} = 279 \pm 17 \pm 20 \text{ MeV}.$$

Both measurements confirm our results, at the 10% level. Many of our uncertainties cancel in the ratio:

$$R_{d/s} = 0.786 \pm 0.042 \text{ [lattice QCD]},$$
$$R_{d/s} = 0.779 \pm 0.093 \text{ [CLEO/BaBar]},$$

which agree marvelously.

*Quenching:* The plot at right shows how $f_{Ds}$ (which has a mild chiral extrap'n) depends on $n_f$. As expected (?), there is a ~20% shift from $n_f = 0$ to 2 and 2+1.




## Mass of $B_c$ Meson

Ian F. Allison, Christine T.H. Davies, Alan Gray,
Andreas S. Kronfeld, Paul B. Mackenzie, James N. Simone
(HPQCD and Fermilab Lattice Collaborations)
*Phys. Rev. Lett.* **94**, 172001 (2005) [arXiv:hep-lat/0411027].

The $B_c$ meson consists of a bottom quark and a charmed antiquark. It was first observed by CDF during Run I of the Tevatron. The decay mode was $B_c \to J/\psi l\nu$, the neutrino was missed, so the mass resolution was ± 400 MeV. DØ confirmed the observation in Run 2, also in semileptonic decay.

From *B Physics at the Tevatron: Run II and Beyond* [hep-ph/0201071], it was clear that nonleptonic modes would be much, much better.

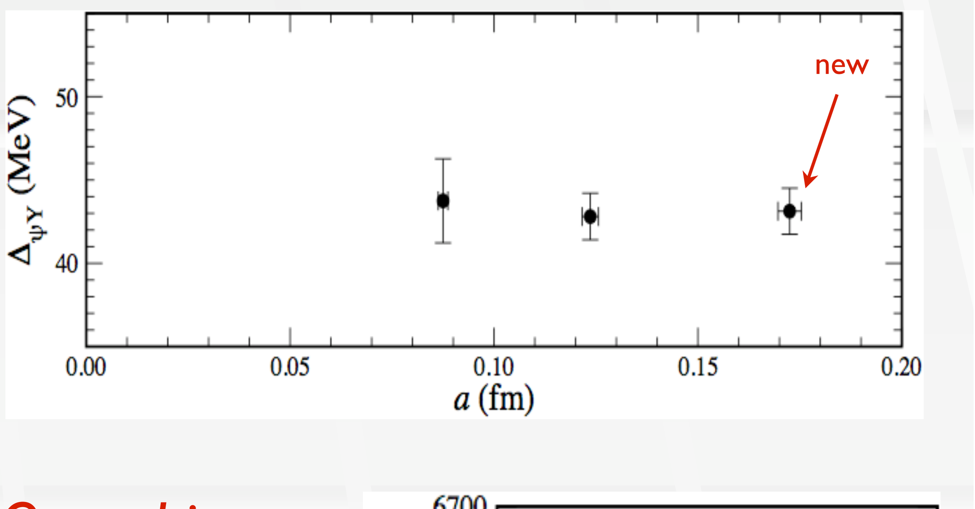
At Lattice 2004, we presented results that were in almost final form. By mid-November, we posted our paper on the arXiv:

$$m_{Bc} = 6304 \pm 12 \,^{+18}_{-0} \text{ MeV}.$$

Later, CDF presented evidence for $B_c \to J/\psi\pi$ decay, reconstructing a mass [hep-ex/0505076]

$$m_{Bc} = 6287 \pm 5 \text{ MeV}.$$

Our result is based on computing the mass splitting $\Delta_{\psi\Upsilon} = m_{Bc} - (\bar{m}_\psi + m_\Upsilon)/2$, which is astonishingly flat as a function of lattice spacing:




*Quenching:* The plot at right shows how $m_{Bc}$ and the splitting change with $n_f$. The quenched result on its own would be far off.