High-Precision Numerical QCD

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Particle Physics Methodology

The Curse: High Precision

Eg) Quantum Electrodynamics (QED) $\Rightarrow$ magnetic moment of e:

$$\frac{g_e}{2} = 1.000579826087\ldots \quad \text{(QED Theory)}$$
$$= 1.000579826087\ldots \quad \text{(Experiment)}$$

12 digits $\equiv$ NYC–LA distance to few nanometers.

Eg) Standard Model of Particle Physics

- QCD (nuclear) + QED + Weak (radioactivity) interactions of quarks, leptons . . . .
- Extremely successful but incomplete ($\Rightarrow$ LHC).
- Major experimental program (Cornell, Fermilab, KEK, SLAC . . .) to push Standard Model to failure:
    - ◇ Compare high-precision measurements of radioactive decays of heavy quarks (c and b) with theory.

    $\Rightarrow$ Need high-precision (1–2%) QCD since quarks bound in mesons.

The Problem: Quantum Field Theory (QCD)

1) Nonlinear: e.g.,

$$\nabla \cdot \mathbf{E} = g(\mathbf{A} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{A}) + \cdots$$

matrix field

2) Strongly coupled:

$$\alpha_s \equiv \frac{g^2}{4\pi\hbar c} \approx 1$$

$\Rightarrow$ nonperturbative!

N.B. For QED:

$$\alpha_{\mathrm{QED}} \approx \frac{1}{137.04}$$

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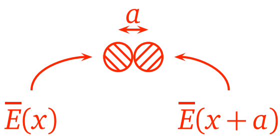
3) Structure at all scales:

Eg) Electric field averaged over probe size $a \Rightarrow$



$$\left\langle \left(\overline{E}(x) - \overline{E}(x+a) \right)^2 \right\rangle \to \frac{1}{a^4} \quad \text{as } a \to 0$$

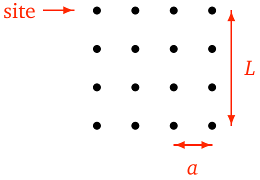
$\Rightarrow$ $\mathbf{E}(\mathbf{x}, t)$ is infinitely rough at short distances!

$\Rightarrow$ Structure at arbitrarily short distances!

Numerical QCD

Lattice Approximation

Continuous
Space & Time $\longrightarrow$



$\Rightarrow$ QCD $\rightarrow$ multidimensional integration:

$$\int \mathscr{D}A_\mu \ldots \mathrm{e}^{-\int L\mathrm{d}t} \longrightarrow \int \prod_{x_j \epsilon \, \mathrm{grid}} \mathrm{d}A_\mu(x_j) \ldots \mathrm{e}^{-a\sum L_j}.$$

$\Rightarrow$ Monte Carlo integration over millions of variables.

$\Rightarrow$ Integrate quarks out $\rightarrow$ $\det(D \cdot \gamma + m)$ (expensive!).

N.B. Quantum Numerical Analysis

Eg)

$$\partial\psi \to \Delta\psi + c(a)\, a^2\, \Delta^3\psi + \cdots$$

where

$$\Delta\psi(x) \equiv \frac{\psi(x+a) - \psi(x-a)}{2a}$$

and

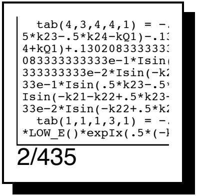
$$c(a) = -\frac{1}{6} + \text{Contribution from } x < a \text{ physics}$$

Numerical Analysis

Theory & context specific
$\Rightarrow$ not universal!

Compute using pert'n theory.
(Asymptotic freedom.)

N.B., vertex rule $ig\gamma_\mu \;\rightarrow\;$ 435 pages of (in LQCD):



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  tab(4,3,4,4,1) = -.
5*k23-.5*k24-kQ1)-.13
4+kQ1)+.1302083333333
083333333333e-1*Isin(
333333333e-2*Isin(-k2
33e-1*Isin(.5*k23-.5
Isin(-k21-k22+.5*k23-
33e-2*Isin(-k22+.5*k2
  tab(1,1,1,3,1) = -.
*LOW_E()*expIx(.5*(-k
```

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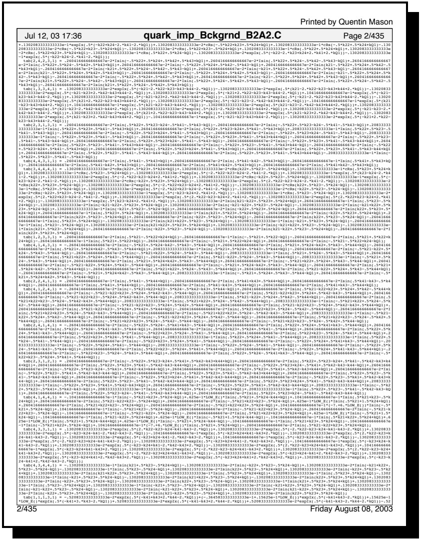

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Two QCD Breakthroughs

1) Larger a (1992)

Before $\Rightarrow$ need $a < 0.05\,\mathrm{fm}$.

Now $\Rightarrow$ $a = 0.1$–$0.4\,\mathrm{fm}$ works.

Simulation cost $\propto (1/a)^6$

$\Rightarrow$ new simulations cost 10^2–10^6 times less!

2) Smaller u/d Quark Masses (2000)

Before $\Rightarrow$ $m_{u/d}$ 10–20$\times$ too big; vac. pol'n impossible.

Now $\Rightarrow$ 3–5$\times$ smaller masses; extrapolate to real QCD.

Vac. pol'n enters at 15–30%

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Does it work?

LQCD Confronts Experiment

- Choose bare α_s then tune 5 free (bare) parameters to match experimental data:

$$\begin{aligned} m_u = m_d &\longleftrightarrow m_\pi^2 \\ m_s &\longleftrightarrow 2m_K^2 - m_\pi^2 \\ m_c &\longleftrightarrow m_{\eta_c} \\ m_b &\longleftrightarrow m_\Upsilon \\ a &\longleftrightarrow m_{\Upsilon'} - m_\Upsilon \end{aligned}$$

  - ◇ Tunings decouple.
  - ◇ Experimental errors negligible.
  - ◇ Residual isospin, e/m errors negligible.

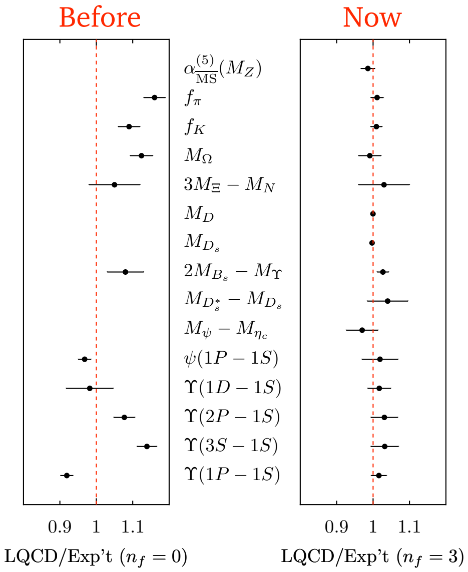
- No other free parameters.

- Test by: a) computing lots of other quantities and comparing with experiment; b) symmetry restoration.

Davies et al, Phys. Rev. Lett. 92:022001, 2004. (HPQCD, MILC, Fermilab, UKQCD)
Mason et al, Phys. Rev. Lett. 95:052002,2005. (HPQCD, UKQCD)
Follana et al, Phys. Rev. D75:054502, 2007. (HPQCD, UKQCD)

Lattice QCD/Experiment

(No free parameters!)



Tests:

- $m_{u,d}$ extrapolation;
- masses and wavefunctions;
- s quark;
- light-quark baryons;
- light-heavy mesons;
- heavy quarks (no potential model. . .);
- improved staggered quark vacuum polarization.

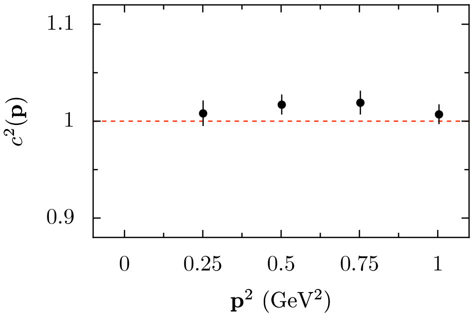
⇒ Most accurate strong interaction calculation in history!

Symmetry Restoration

Eg) Lorentz invariance requires

$$c^2(\mathbf{p}) \equiv \frac{E^2(\mathbf{p}) - m^2}{\mathbf{p}^2} = 1 \qquad \text{for all } \mathbf{p}.$$

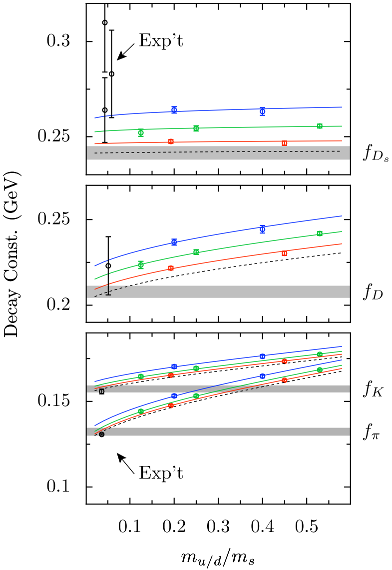
For η_c when $m_c = 0.67/a$:



N.B. Drop a^2 correction $\Rightarrow$ $c^2 = 0.56(1)$.

High Precision Now

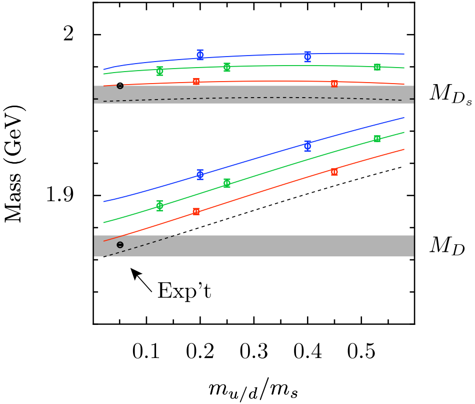
Weak Decay Constants



- HISQ formalism for c, u, d, s (3^{rd} generation staggered quarks).
- Remove $\alpha_s(am)^2(v/c)^0$ and $(am)^4(v/c)^0$ errors.
- Remove one-loop taste-exchange effects.
- a^2 errors 2–3.5× smaller than ASQTAD.
- Treat u, d, s, and c in same fully relativistic formalism.
- Partially conserved axial vector currents ⇒ no renomalization.

Follana et al, arXiv:0706.1726 [hep-lat]. (HPQCD, UKQCD)

D, D_s Masses



Conclusion

Few percent precision ⇒ superb opportunity for lattice QCD to have an impact on fundamental particle physics.

- LQCD essential to high-precision heavy-quark physics at Cornell, Fermilab, KEK, SLAC . . . .
- Predicting experimental results ⇒ much needed credibility for LQCD.
- Low-mass $u/d/s$ essential to high precision.
- Landmark in history of quantum field theory: quantitative verification of nonperturbative technology (c.f., 1950s).
- Ready for beyond the Standard Model, strong-coupling beyond QCD?