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Global gyrokinetic particle simulation of turbulence and transport in realistic tokamak geometry

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Abstract. A general geometry model has been developed for the gyrokinetic toroidal code GTC[1] with a number of highly desirable features including a systematic treatment of plasma rotation and equilibrium $\mathbf{E} \times \mathbf{B}$ flow, realistic plasma profiles and corresponding magnetohydrodanamic (MHD) equilibria. A symmetry coordinate system is used to construct a relatively regular mesh in real space for strongly shaped toroidal plasmas, which also facilitates straightforward visualization. By rescaling the radial coordinate, grid size is correlated with the local gyroradius which may vary substantially from the core to the edge. Gyrokinetic transformation of potential and charge density between particle and guiding center positions in general geometry is carefully treated, taking into account the finite ratio of the poloidal to the total field (B_θ/B). The applied equilibrium $\mathbf{E} \times \mathbf{B}$ flow, which is believed to play an important role in determining the turbulence level, is calculated from our global neoclassical particle code GTC-Neo[2]. In the large aspect ratio circular geometry limit, cross benchmarks with the original GTC code show good agreement in real frequency, growth rate, steady-state heat flux and zonal flow amplitude for the ion temperature gradient driven microinstabilities (ITG modes).

1. Introduction

Understanding turbulence and associated transport in toroidal plasmas [3-5] is one of the key issues in magnetic fusion research. In the past decade as computer resources rapidly increase and advanced numerical algorithms are developed, significant progress has been made with the most prominent being the first-principles based gyrokinetic particle approach [6]. The simulation study carried out by the gyrokinetic toroidal code GTC[1] represents one of the more productive examples. GTC was originally developed to focus on fundamental nonlinear turbulence physics. It is a full-torus global code using a field-line-following mesh and a real space field solver. Global turbulence simulations for toroidal plasmas are highly demanding for the following reasons: (i) the turbulence-generated zonal flow has a typical radial scale as large as the system size, even though turbulence itself is on the much smaller scale of the gyroradius; (ii) the equilibrium $\mathbf{E}\times\mathbf{B}$ shear flow which also plays an important role in determining turbulence levels typically has the large scale size of the plasma minor radius; (iii) turbulence spreading to the linearly stable zone results in nonlocal transport which is a truly global phenomenon. To pose the simplest problem while keeping the important global physics properties, a simplified model was utilized in the GTC simulations, such as simple magnetic geometry with a large aspect ratio circular concentric cross section and the assumption of uniform pressure. This proved to be an effective means of gaining key insights into the complexity of toroidal turbulence system. As a result, GTC simulation has led to a number of important findings with regard to zonal flow effects, transport scaling with device size, turbulence spreading, etc [1,7-9]. While such a simplified model is a necessary and useful tool to separate and clarify fundamental physics issues, more realistic features are needed as the research moves forward. Particularly for simulating turbulence phenomena in tokamak experiments, a more comprehensive model is needed which consistently incorporates the influence of general geometry, realistic plasma profiles, plasma flow, neoclassical equilibrium, Coulomb collisions and other features. In this paper we present such a model with emphasis on the general geometry capability, which has now been implemented into GTC. The general geometry GTC is interfaced with TRANSP, a widely used experimental data analysis software tool for specifying experimental plasma profiles of temperature, density and toroidal angular velocity, and also with various numerical MHD equilibrium codes, including the Jsolver, qsolver and ESC codes. This is a major project within the SciDAC Center for Gyrokinetic Particle Simulation.

2. Coordinate system and mesh construction

Magnetic flux coordinates in which the radial coordinate labels magnetic surfaces are generally used for a toroidal system and associated with MHD equilibria. Our gyrokinetic simulation in principle can use arbitrary flux coordinates with straight field lines. In the flux coordinates, the field-line-following mesh, which possesses the highest efficiency by capturing the flute-type character of the drift wave turbulence in toroidal plasma, can be easily constructed. A preferable flux coordinate can be chosen in terms of a different requirement. A symmetric coordinate system in which the toroidal angle φ is chosen to be the azimuthal angle of cylindrical coordinates is preferable in many cases. These coordinates are relatively uniform compared to other which have been previously used, and advantageous for constructing a relatively regular mesh in real space for strongly shaped plasmas. It also facilitates straightforward visualization with the poloidal plane defined with the physical angle φ. The radial coordinate is defined as $r = \sqrt{\psi/\psi_e}$ where ψ and ψ_e are the toroidal flux and its value on the last magnetic surface, respectively. This same radial coordinate is widely used in the experimental community.

Because of the flute-type character of drift wave turbulence in toroidal plasmas, with $k_\parallel \ll k_\perp$, where $k_\parallel$ and $k_\perp$ are the parallel and perpendicular wave numbers, respectively, GTC uses the field-line-following mesh which shows high efficiency for calculating the turbulent field. For drift wave turbulence, the spatial scale length in the perpendicular direction is generally in correlation with the local gyroradius $\rho_j \propto \sqrt{T_j}$, which may vary substantially from the core to the edge of the plasma. For instance, it is common in National Spherical Torus (NSTX) plasmas that the ion temperature changes from $\sim KeV$ in the core to $\sim 10eV$ near the separatrix region at the plasma edge. Therefore, for a global simulation, which includes the entire radial domain, it is important to use a nonuniform grid with grid size in the perpendicular direction correlated with the local gyroradius for improved spatial resolution and efficiency. To this end, we re-scale the radial coordinate by defining ρ as follows:

$$\frac{d\rho}{dr} = \sqrt{T_c/T_i(r)}, \tag{1}$$

where T_c is the temperature at a reference radial location. Working with the new coordinate ρ, we use an even spaced radial grid, which offers great convenience for frequent operations such as particle sorting, charge deposition, gathering, etc. This allows the grid size in real space to be correlated with the local gyroradius: $\Delta r \sim \sqrt{T_i(r)/T_c}$. In the poloidal direction, the grid size $\Delta\theta(r)$ is uniform on a flux surface, while varying over different flux surfaces. The grid size $\Delta\theta(r)$

is determined so as to make the poloidal arc length Δl_θ near the mid-plane correlated with ρ_i. An example of such a grid on the toroidal plane $\varphi = 0$ is shown in Fig.1. Generally, a two-dimensional mesh on the $\varphi = 0$ plane is set up first. A three-dimensional mesh is constructed by following each (approximate) field line which starts at a grid point on the $\varphi = 0$ plane and has $\bar{q}(r)\theta - \varphi =$constant, with $\bar{q}$ slightly changed from the usual safety factor $q(r)$ so that the approximate field lines will lead back to one of the grid points on the $\varphi = 0$ plane.

The gyrokinetic particles are followed in general flux coordinates using guiding center Lagrangian equations instead of Hamiltonian equations, which require construction of canonical variables[10] that are complicated forms in general geometry and inconvenient to use. The guiding center Lagrangian obtained by Littlejohn has the following normalized form[10,11]

$$L(\mathbf{x}, \dot{\mathbf{x}}; t) = (\mathbf{A} + \rho_\| \mathbf{B}) \cdot \mathbf{v} - H, \tag{2}$$

with the guiding center Hamiltonian $H = \rho_\|^2 B^2/2 + \mu B + \Phi$. Here, the magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$, $\rho_\| = v_\|/B$ is the parallel gyroradius, μ is the magnetic moment, and Φ is the electric potential. The independent variables are $\mathbf{x} = (r, \theta, \varphi, \rho_\|)$. The particle guiding drift motion is governed by the Lagrangian equation

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{x}_i} L\right) - \frac{\partial}{\partial x_i} L = 0. \tag{3}$$

The obtained equations for $d\mathbf{x}/dt$ are suitable for any generalized flux coordinates.

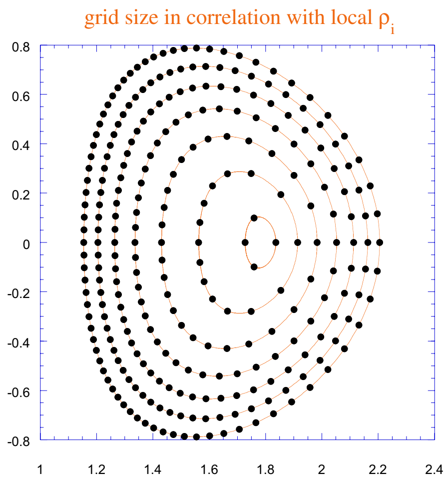


Figure 1. An example of nonuniform grid on a poloidal plane showing grid size in correlation with local ion gyroradius.

3. Simulation results

In this section we present new scientific results from our present studies. Details on the form of the basic equations and the associated gyrokinetic transformation in general geometry will be presented in a separate publication.

3.1. Benchmarks

The general geometry version of GTC has been carefully benchmarked, in the large aspect ratio circular geometry limit, against the original GTC code, which uses a simple analytical MHD equilibrium. For this benchmark, a corresponding numerical equilibrium is produced for the general geometry GTC. The numerical equilibrium includes a small *Shafranov* shift due to non-zero plasma beta and higher order (in small inverse aspect ratio) corrections, which are neglected in the analytical equilibrium. The benchmarks are carried out for ion temperature gradient

(ITG) modes with a simplified adiabatic response for the electron dynamics. The representative parameters for the familiar cyclone case[12] are used here: inverse aspect ratio $a/R_0 = 0.358$, ion temperature profile $R_0/L_T = 6.92 \exp\{-[(r-0.5)/0.28]^6\}$, $T_e/T_i = 1$, $q = 0.854 + 2.184r^2$, and $L_T/L_n = 0.319$.

The linear benchmark simulations are carried out in a radial domain from $0.2a$ to $0.8a$, and the ITG instability is measured at $r = 0.5$ where the temperature gradient is peaked. As illustrated in Fig.2, good agreement is obtained for the real frequency ω_r, while the growth rate γ of the higher-n modes from the new general geometry version of GTC are slightly lower. The overall difference in frequency magnitude $|\omega|$ is less than 5%. The contour plots of the electrostatic potential perturbation on a poloidal plane show quite similar eigenmode structures from the two simulations.

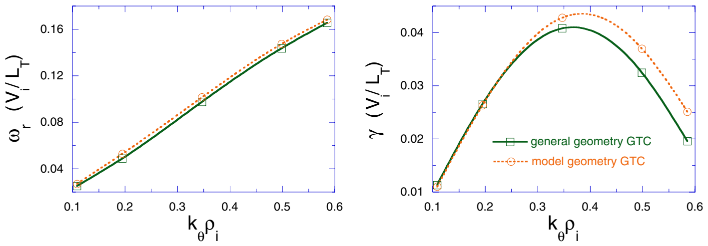


Figure 2. Real frequency ω_r (left) and growth rate γ (right) of ITG instability versus poloidal wave number k_θ, compared with the original GTC calculation.

In the nonlinear ITG benchmark, the velocity space nonlinearity is included, which may have considerable effect on turbulence dynamics[13]. Flat temperature and density profiles are used in the general geometry GTC for benchmarks in order to be consistent with the original GTC. The radial simulation domain here is from $r = 0.1a$ to $r = 0.9a$. As shown in Fig.3, the nonlinear benchmark results are in good agreement for both steady state heat flux and zonal flow over the entire radial domain. Some difference is found in the linear phase and in the approach to steady state. It is also found that the self-generated zonal flow in ITG turbulence has a global scale of order the minor radius, with a roughly odd parity. In the Fig.3 the energy flux, which is the sum of the heat flux and the convective flux due to the particle flux, is also plotted. The result that the energy and heat flux are right on the top of each other indicates that no particle flux is produced – a well known fact about electrostatic turbulence with adiabatic electrons. It represents a rigorous test for such a complex simulation.

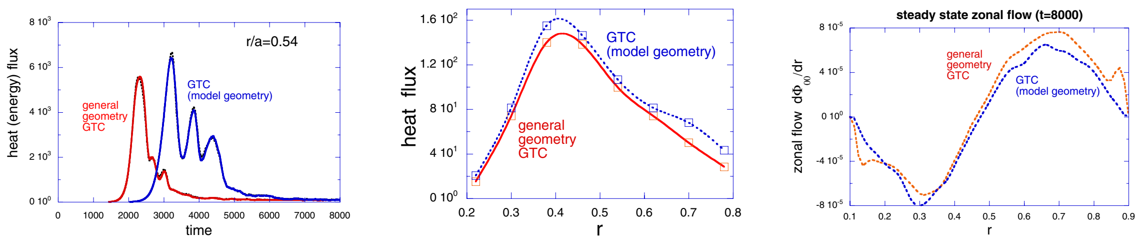


Figure 3. Time history of ion heat flux at $r = 0.54$ (left), steady state heat flux profile (middle) and zonal flow profile (right) of nonlinear ITG simulation with simple magnetic geometry, compared with the original GTC result.

3.2. Applications

Most linear analyses of microinstability are carried out locally, neglecting the radial variation of equilibrium quantities such as pressure gradient and pressure itself. While GTC is a global toroidal code, in order to focus on the simplest problems involving shear effects due to the radial variation of the pressure gradient, plasma temperature and density were assumed constant in the original simulations. While such a treatment appears necessary and useful to separate and clarify fundamental physics issues, it does not realistically capture the comprehensive global physics. In Fig.4 we present an example of global ITG instability from the general geometry GTC simulation, taking into account the radial variation of temperature and density consistent with their gradient profiles. Compared to the simulation with constant T and n, the ITG growth rate is considerably reduced, with no significant change in the real frequency. Meanwhile, the contour plot of the electric potential on a poloidal plane shows that the eigenmode structure is twisted in the poloidal direction.

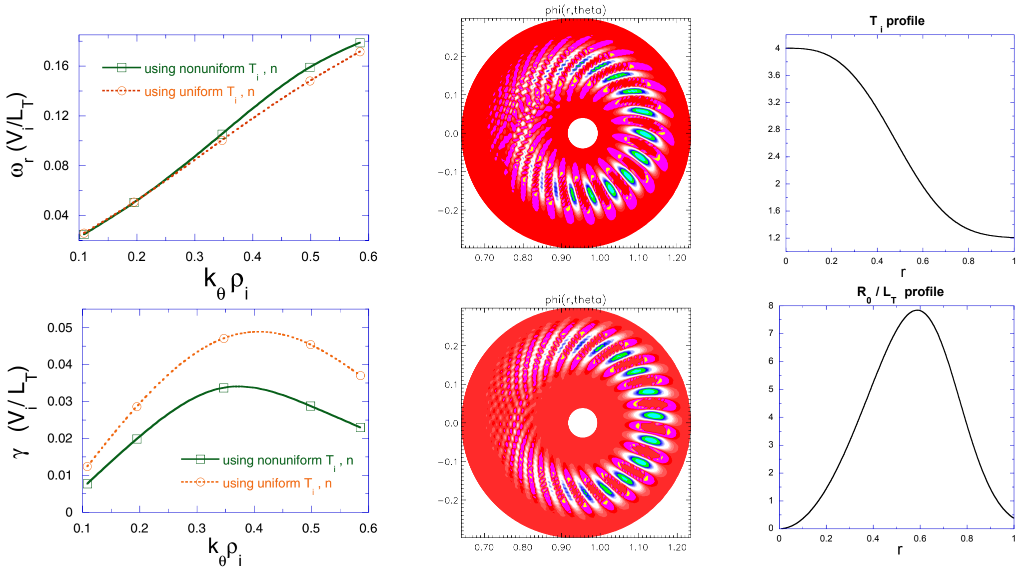


Figure 4. Simulation results of a nonlocal ITG instability using radially varying temperature and density, compared with the simulation with uniform temperature and density: real frequency ω_r (upper-left) and growth rate γ (lower-left) vs $k_\theta \rho_i$; contour plot of electric potential from simulation with nonuniform (upper-middle) T and n and uniform T and n (lower-middle); ion temperature profile (upper-right) and corresponding gradient profile(lower-right) used in the simulations.

We have also applied our new nonlinear ITG simulation capability to a shaped toroidal plasma, the DIII-D experiment at General Atomics, San Diego, with the same model profiles as in Fig.4. The snapshots of the turbulence intensity profile are plotted in Fig.5, which shows that the turbulence, which grows initially in the linearly unstable region, spreads in both the inward and outward radial directions into the stable regions, leading to radially global turbulence and transport nonlocality. Also presented are three snapshots of the electric potential contour plot on a poloidal plane, which illustrate the dynamic process of turbulence development: in the earily phase the elongated radial streamer structure of the eigenmode is linearly driven in the localized unstable zone, then is broken up as self-generated zonal flow is established, and eventually envolves into widely spread global turbulence.

4. Summary

We have developed a more realistic model for simulating plasma turbulence in actual tokamak experiments. It incorporates the comprehensive influence of general geometry, realistic plasma profiles, plasma rotation, neoclassical (equilibrium) electric field, and Coulomb collisions. The

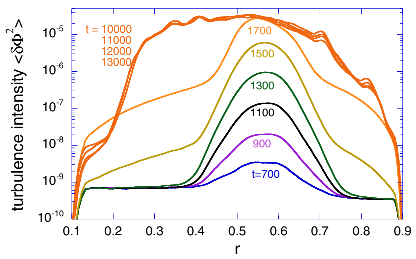
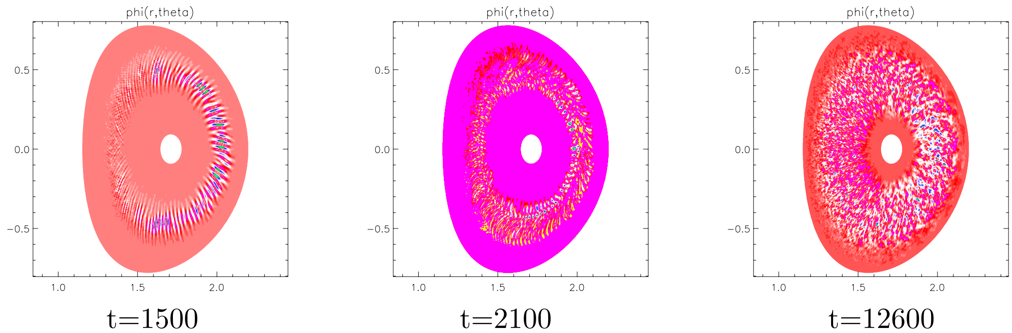


Figure 5. Snapshots of turbulence intensity profile (left), and contour plot of electric potential on a poloidal plane (bottom) from a simulation of a shaped plasma with typical DIII-D parameters.



careful benchmarks of the linear and nonlinear characters of the well known ion temperature gradient instability have been carried out to validate the new capability. Application of this code to DIII-D sized shaped plasma has produced interesting results with regard to nonlinear dynamics of ITG turbulence spreading and associated energy cascading to the longer wavelength (low-n) modes. This highly desirable capability makes GTC a powerful tool to investigate the properties of plasma transport and confinement of actual tokamak devices. Since finite beta (ratio of plasma pressure to magnetic pressure) toroidal plasma electromagnetic effects become more important in determining the transport level in burning plasma experiments such as ITER, further extensions of the GTC project to simulate electromagnetic turbulence are currently being actively pursued.

Acknowledgments
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