Solver Challenges in Adaptive Mesh Refinement Ice Sheet Modeling

Dan Martin

Lawrence Berkeley National Laboratory

Copper Mountain

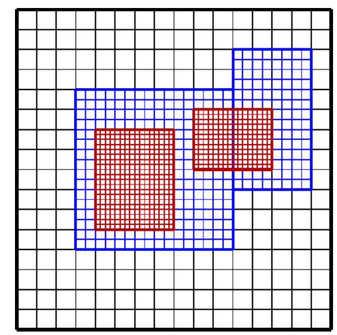
March 21, 2016

Joint work with:

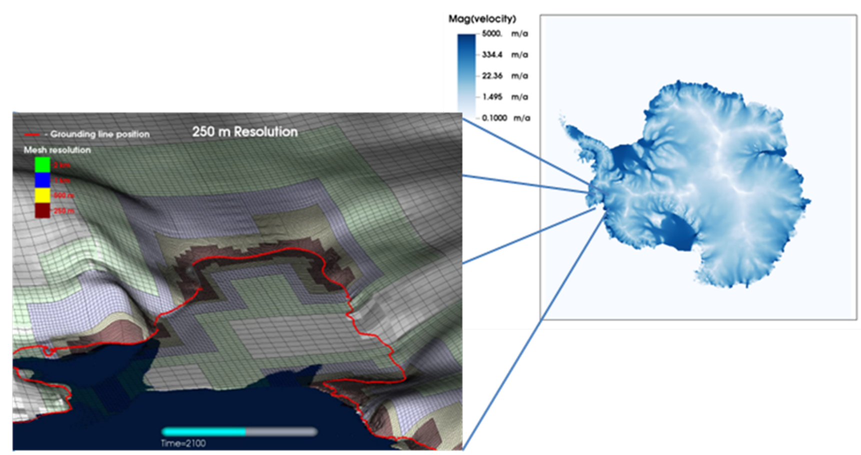
- Mark Adams (LBNL)
- Xylar Asay-Davis (Potsdam-PIK)
- Stephen Cornford (Bristol)
- Stephen Price (LANL)
- Doug Ranken (LANL)
- Esmond Ng (LBNL)
- William Collins (LBNL)

BISICLES Ice Sheet Model

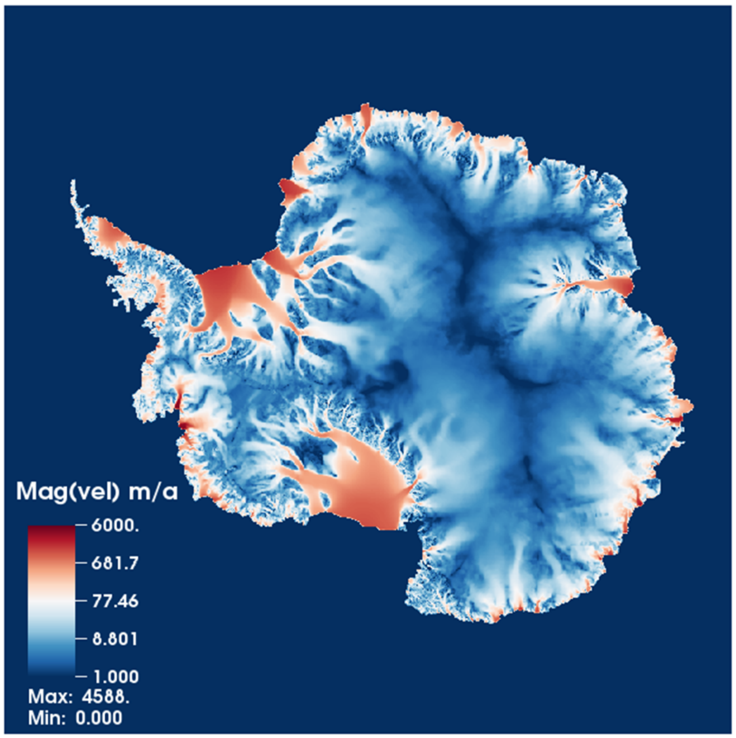
- Scalable adaptive mesh refinement (AMR) ice sheet model
  - Dynamic local refinement of mesh to improve accuracy
- Chombo AMR framework for block-structured AMR
  - Support for AMR discretizations
  - Scalable solvers
  - Developed at LBNL
  - DOE ASCR supported (FASTMath)



- Collaboration with Bristol (U.K.) and LANL
- Variant of “L1L2” model (Schoof and Hindmarsh, 2010)
- Coupled to Community Ice Sheet Model (CISM).
- Users in Berkeley, Bristol, Beijing, Brussels, and Berlin…

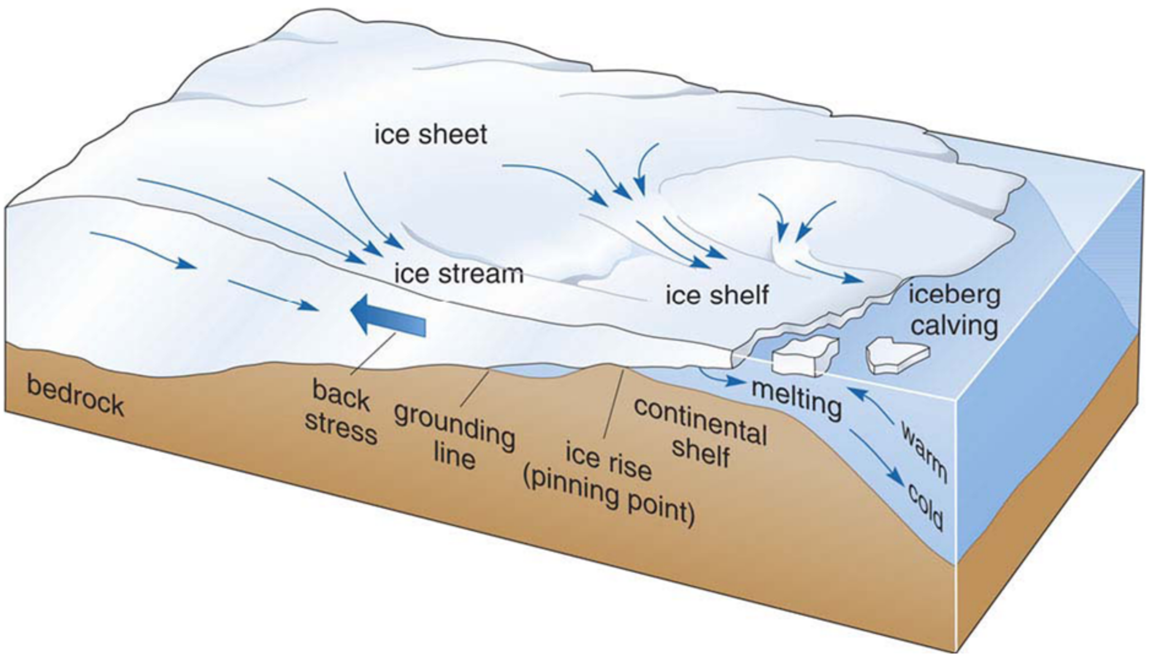


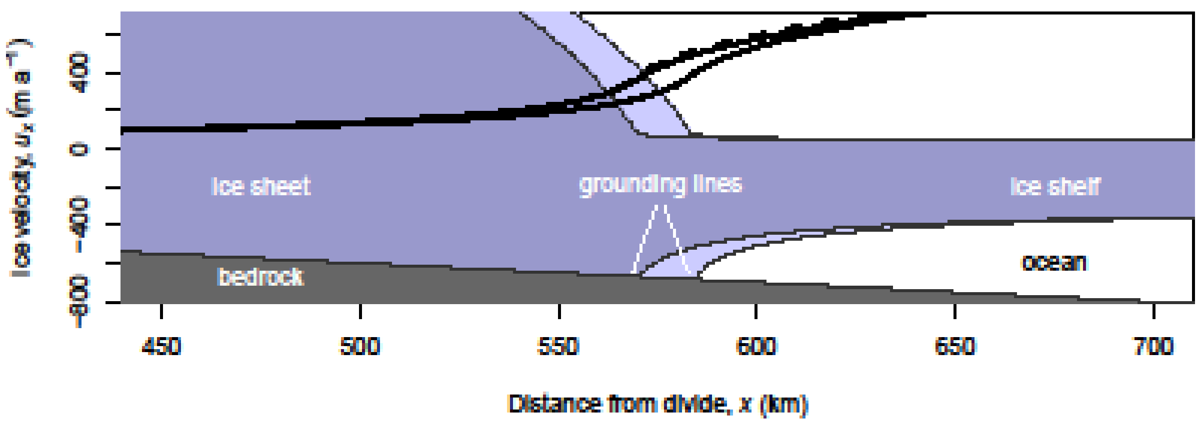
Why is this useful? (another BISICLE for another fish?)



- Ice sheets -- Localized regions where high resolution needed to accurately resolve ice-sheet dynamics (500 m or better at grounding lines)
- Antarctica is really big – too big to resolve at that level of resolution.
- Large regions where such fine resolution is unnecessary (e.g. East Antarctica)
- Well-suited for adaptive mesh refinement (AMR)
- Problems still large: need good parallel efficiency
- Dominated by nonlinear coupled elliptic system for ice velocity solve: good linear and nonlinear solvers

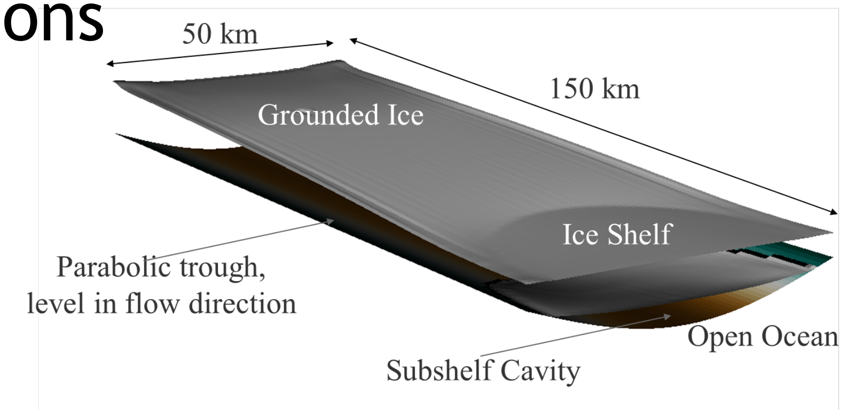
Marine Ice Sheets



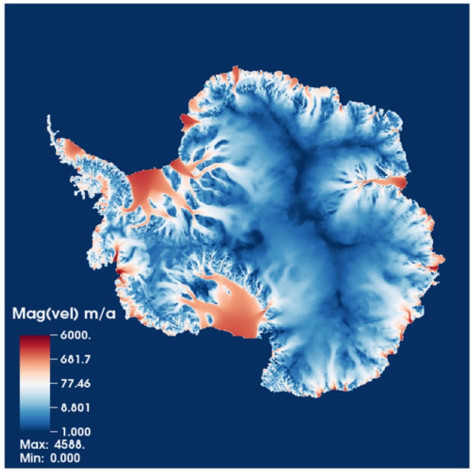


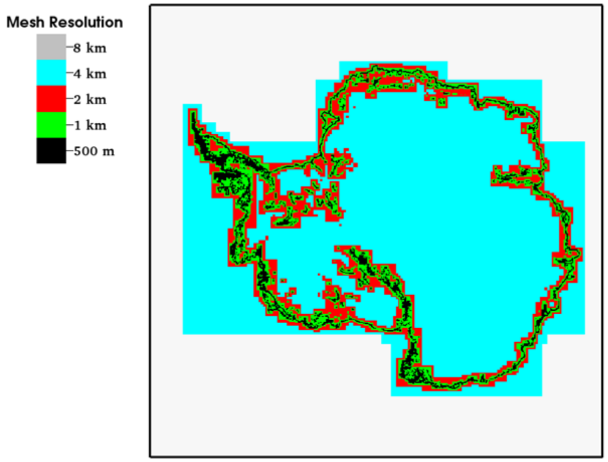
Target Problems

- Idealized Ice-Ocean interaction test problems
  - Simple/small geometries designed to understand GL dynamics and ice-ocean interactions
  - MISMIP3D
  - MISMIP+
  - MISOMIP



- Realistic full-scale
  - Fully-resolved (500m) full-continent
  - Antarctica





BISICLES: Models and Approximations

Physics: Non-Newtonian viscous flow: $\mu(\epsilon\uparrow 2\ ,T) = A(T)(\epsilon\uparrow 2\ )\uparrow$ $(1-n)/2$

Where $\epsilon\uparrow 2$ is the strain rate invariant, typically $n=3$

- **Full-Stokes**
  - Best fidelity to ice sheet dynamics
  - Computationally expensive (full 3D coupled nonlinear elliptic equations)
- **Approximate Stokes**
  - Use scaling arguments to produce simpler set of equations
  - Common expansion is in ratio of vertical to horizontal length scales $(\varepsilon=[h]/[l])$
  - E.g. Blatter-Pattyn (most common "higher-order" model), accurate to O($\varepsilon\uparrow 2$)
  - Still 3D, but solve simplified elliptic system (e.g. 2 coupled equations)
- **Depth-integrated**
  - "Shallow Ice" and "Shallow-Shelf" approximations (accurate to O(ε))
  - Special case of approximate Stokes with 2D equation set

"L1L2" Model (Schoof and Hindmarsh, 2010).

- Uses asymptotic structure of full Stokes system to construct a higher-order approximation
  - Expansion in $\varepsilon = [H]/[L]$ and $\lambda = [\tau_{shear}]/[\tau_{normal}]$ (ratio of shear & normal stresses)
    - Large λ: shear-dominated flow
    - Small λ: sliding-dominated flow
  - Computing velocity to $O(\varepsilon^2)$ only requires τ to $O(\varepsilon)$
- Computationally **much** less expensive -- enables fully 2D vertically integrated discretizations. (can reconstruct 3d)
- Similar formal accuracy to Blatter-Pattyn $O(\varepsilon^2)$
  - Recovers proper fast- and slow-sliding limits:
    - SIA ($1 \ll \lambda \leq \varepsilon^{-1/n}$) -- accurate to $O(\varepsilon^2 \lambda^{n-2})$
    - SSA ($\varepsilon \leq \lambda \leq 1$) – accurate to $O(\varepsilon^2)$

"L1L2" Model (Schoof and Hindmarsh, 2010), cont.

- Can construct a computationally efficient scheme:

1. Approximate constitutive relation relating $grad(u)$ and stress field τ with one relating $grad(u|_{z=b})$, vertical shear stresses τ_{xz} and τ_{xz} given by the SIA / lubrication approximation and other components $\tau_{xx}(x,y,z)$, $\tau_{xy}(x,y,z)$, etc

2. leads to an effective viscosity $\mu(x,y,z)$ which depends only on $grad(u|_{z=b})$ and $grad(z_s)$, ice thickness, etc

3. Momentum equation can then be integrated vertically, giving a nonlinear, 2D, elliptic equation for $u|_{z=b}(x,y)$

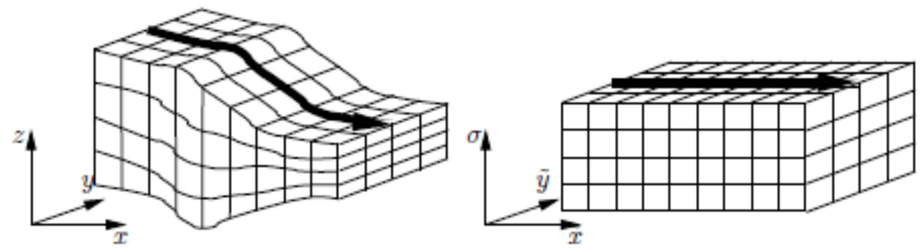
4. $u(x,y,z)$ can be reconstructed from $u|_{z=b}(x,y)$

Modified "L1L2" Model (SSA)*

- Can construct a computationally efficient scheme:
  1. Approximate constitutive relation relating $grad(u)$ and stress field τ with one relating $grad(u|_{z=b})$, vertical shear stresses τ_{xz} and τ_{xz} given by the SIA / lubrication approximation and other components $\tau_{xx}(x,y,z)$, $\tau_{xy}(x,y,z)$, etc
  2. leads to an effective viscosity $\mu(x,y,z)$ which depends only on $grad(u|_{z=b})$ and $grad(z_s)$, ice thickness, etc
  3. Momentum equation can then be integrated vertically, giving a nonlinear, 2D, elliptic equation for $u|_{z=b}(x,y)$
  4. ~~$u(x,y,z)$ can be~~ [illegible] ~~(x,y)~~
  4. Use $u(x,y,z) = u|_{z=b}(x,y)$ (neglect vertical shear in flux velocity)

Discretizations

- Baseline model:
  - Logically-rectangular grid, obtained from a time-dependent uniform mapping.
  - 2D equation for ice thickness



$$\frac{\partial H}{\partial t} = b - \nabla \cdot H\bar{\mathbf{u}}$$

  - Vertically-integrated momentum balance results in 2D nonlinear viscous tensor solve (viscosity a function of velocity) for velocity u_b at the base of the ice:

$$\beta^2 u_b + \nabla\cdot[\mu(\varepsilon^2)(\nabla + \nabla^T)u_b - 2\mu(\nabla\cdot u_b)] = -g/\rho \; H \nabla s$$

β^2 = friction coefficient, ε = strain rate invariant of ice velocity, g = gravity, ρ = ice density, H = ice thickness, ∇s = horizontal gradient of upper surface

  - Enthalpy formulation (Aschwanden) for energy

Discretizations, cont

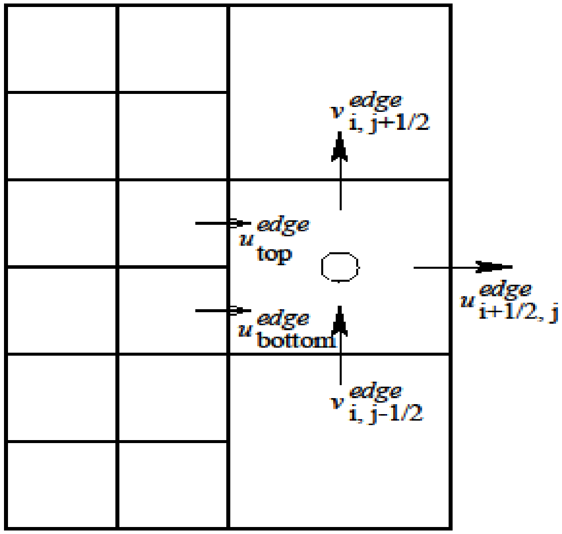
- Use of Finite-volume discretizations (vs. Finite-difference discretizations) simplifies implementation of local refinement.

- Software implementation based on constructing and extending existing solvers using the Chombo libraries.

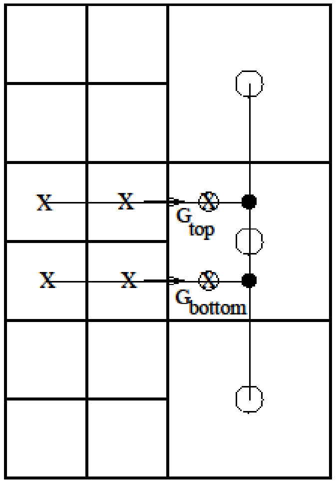
AMR and Finite-Volumes

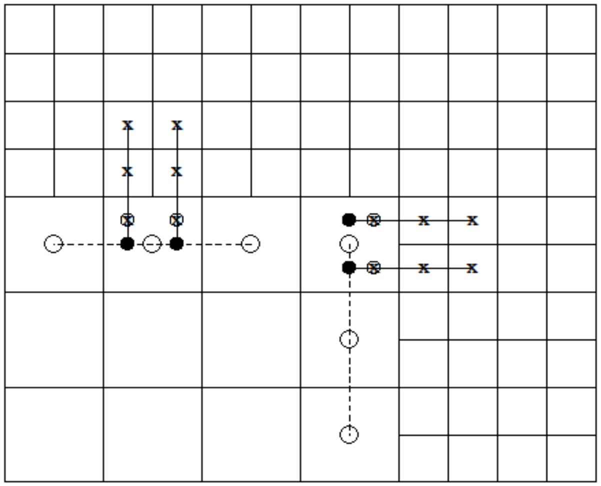
- AMR and finite-volume discretizations play well together:

$$L(\varphi) = \nabla \cdot F$$



- Complex discretizations at coarse-fine interfaces:

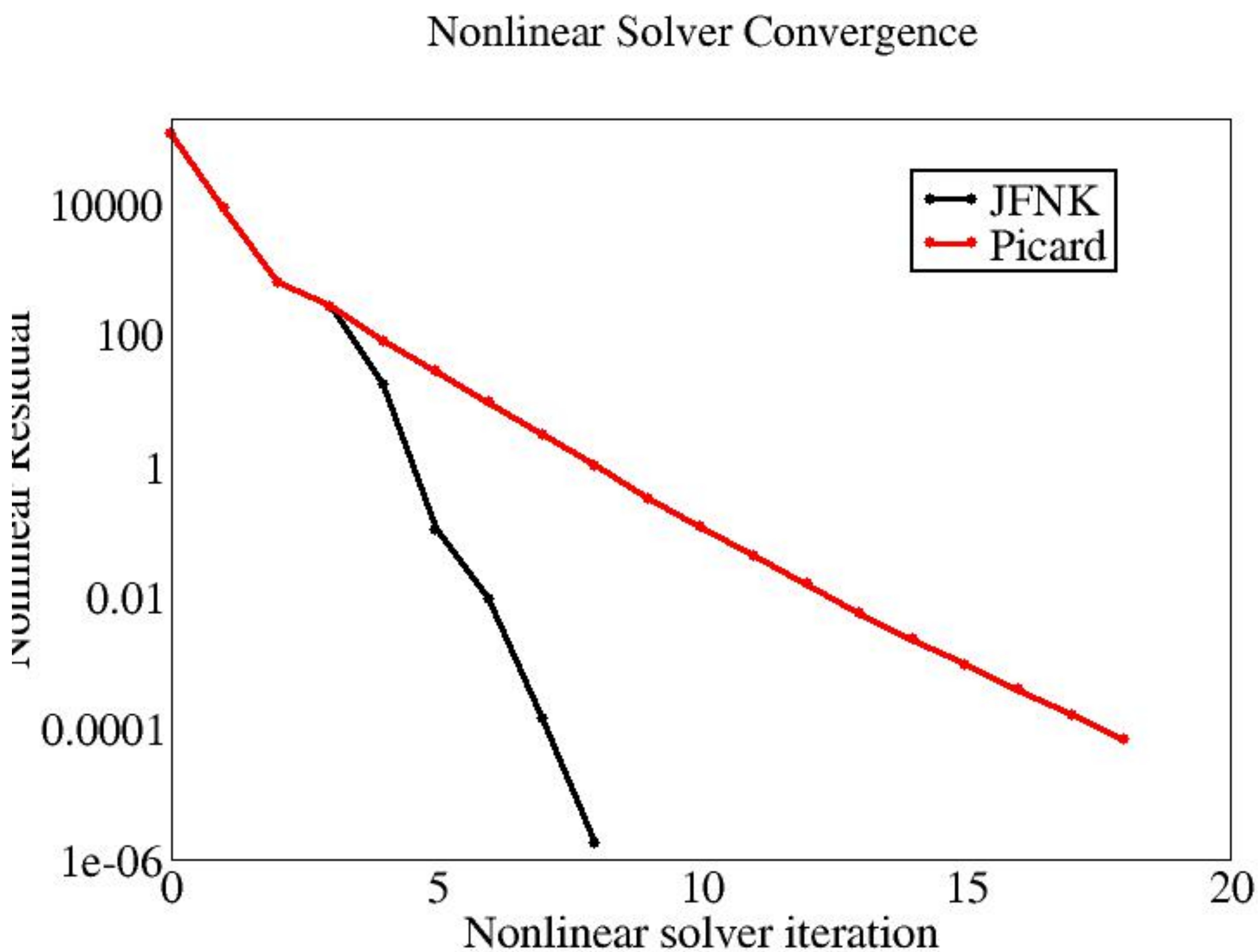




Nonlinear Solvers

- Most computational effort spent in nonlinear ice velocity solve.
- Picard iteration:
  - Robust
  - Simple to implement
  - Slow (but steady) convergence
- Jacobian-free Newton-Krylov (JFNK):
  - More complex to implement
  - Works best with decent initial guess
  - Rapid convergence
  - Well-suited for Chombo AMR elliptic solvers
- Approach – Picard iteration initially, switch to JFNK when convergence slows

Nonlinear Solvers (cont)



Initialization

- Glen's Law singularity ([illegible] [illegible] [illegible] as $\epsilon^2 \to 0$)
  - Regularization: use $(\epsilon^2 + \delta)$ in Glen's law $(\delta = O(10^{-12}))$

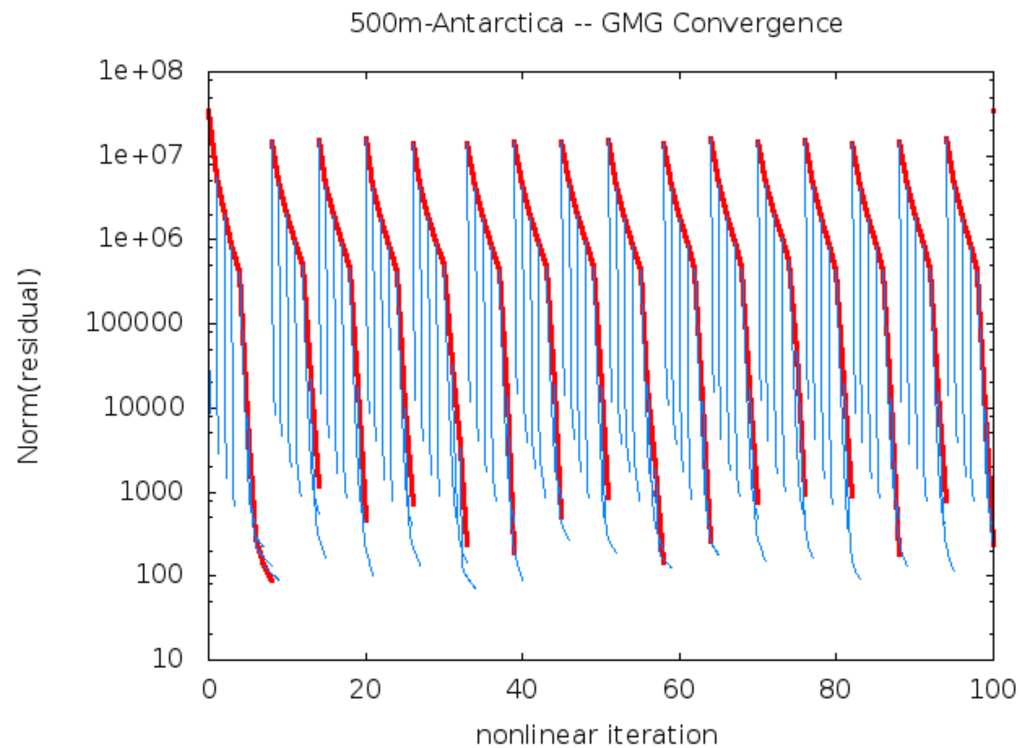
- Natural initial guess (u=0) too close to singularity
  - Takes a lot of solver effort/iterations to push solution away from singularity
  - Better idea – initial linear (constant [illegible]) solve
    - Relatively inexpensive
    - Pushes solution away from singularity -- reasonable initial guess for nonlinear solve
    - (equivalent to homotopy/regularization with a large [illegible])
- Natural form of grid sequencing from AMR

Well-posedness

- Disconnected floating ice (icebergs) are ill-posed
- Can emerge through Ice dynamics (ice-ocean coupling)
- Can also emerge from AMR regridding – interpolation
- Solution – sweep domain for disconnected ice
  - "Marching" scheme (start with grounded ice and march outward in repeated passes
  - AMR-aware connected-components scheme (scalable)
    - (Zou, Martin, et al (2015))

Linear Solvers

- Need good linear solver performance!
- Chombo native solvers – Geometric MultiGrid (GMG)
  - Follows Naturally from AMR hierarchy
- When it works, it works really well (after some tuning)
  - Matrix-free!
  - Relatively efficient
- Works well enough on some problems (Full-continent Antarctica!)



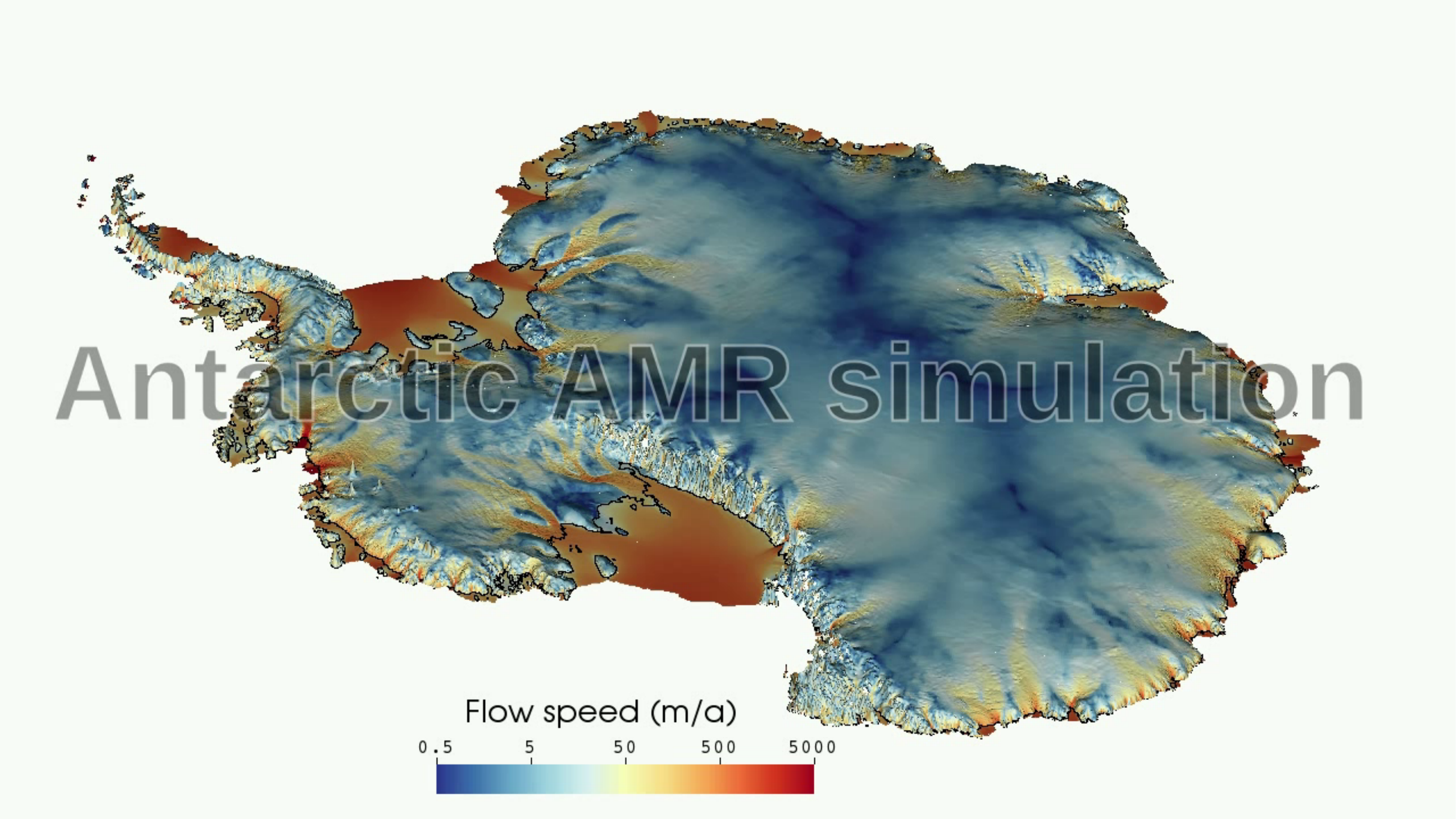
Example – 1000-year Antarctic simulations

- Range of finest resolution from 8 km (no refinement) to 500m (4 levels of factor-2 refinement)

- At initial time, subject ice shelves to extreme (outlandish) melting:
  - No melt for h < 100m
  - Range up to 800m/a where h > 400m.
  - **No melt applied in partially-grounded cells**

- For each resolution, evolve for 1000 years
- Solver tolerances relatively loose

Results:



U.S. DEPARTMENT OF ENERGY | Office of Science
BISICLES
Berkeley Lab
Los Alamos National Laboratory EST. 1943
University of Bristol

Linear Solvers, cont.

- ***However*** – GMG sometimes fails (stalls, diverges)
  - Stalling – finer than 1 km resolution
  - Failure – where Ice shelves are important (Ice/ocean)

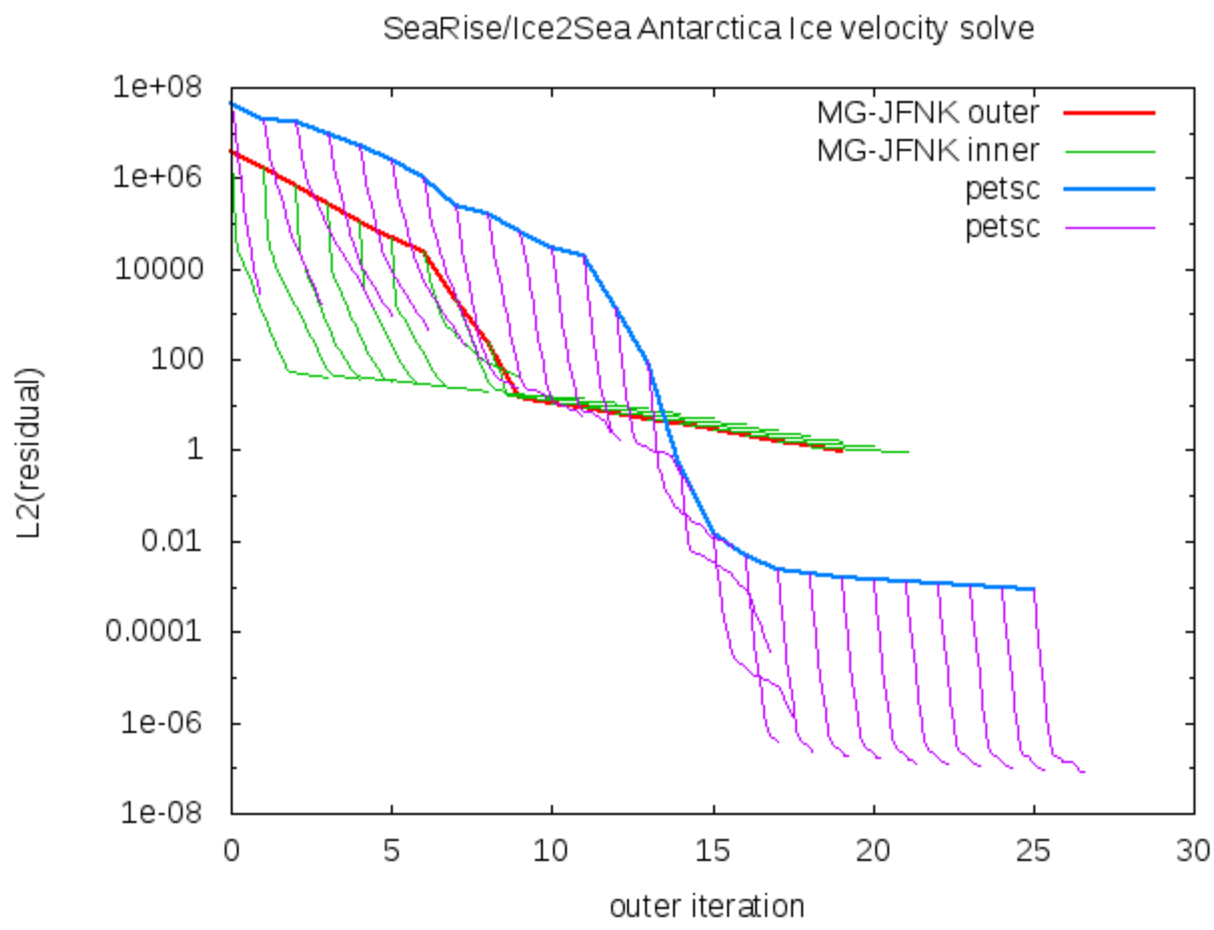
- Sharp/strong coefficient gradients (hard to coarsen)

- **β^2 ⌧ 0 (Change from Parabolic ⌧ Elliptic) at GL:**

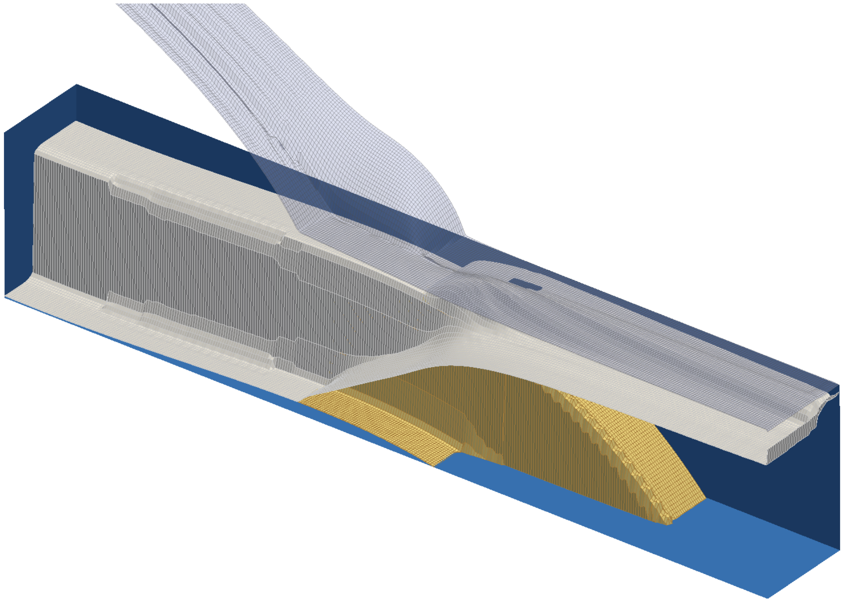
$$\beta^2 u_b + \nabla\cdot[\mu(\varepsilon^2)(\nabla + \nabla^T)u_b - 2\mu(\nabla\cdot u_b)] = -g/\rho \; H \nabla s$$

- **Solution - AMG (PETSc gamg/Hypre BoomerAMG)**

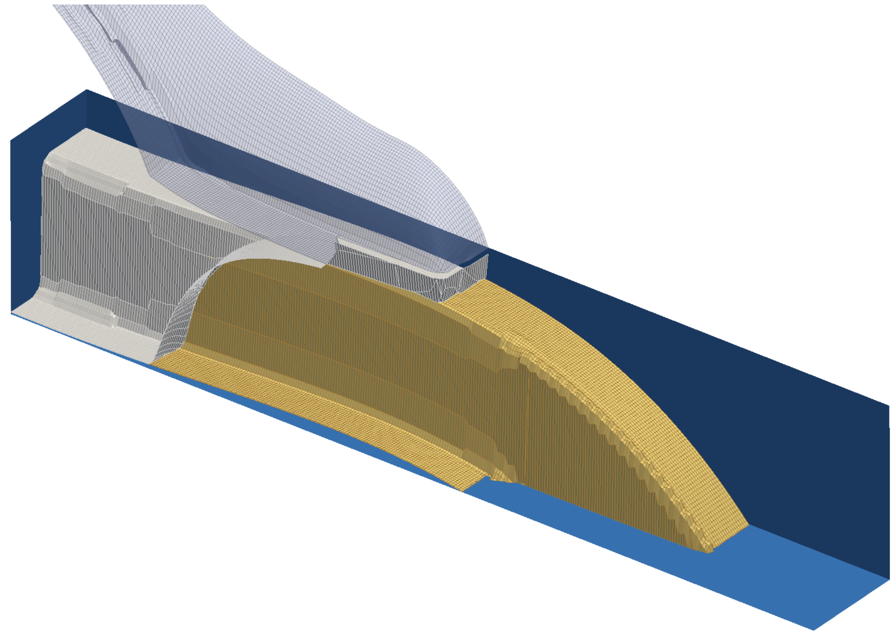
Linear Solvers – GAMG vs. Geometric MG



MISOMIP (Asay-Davis et al (2015))



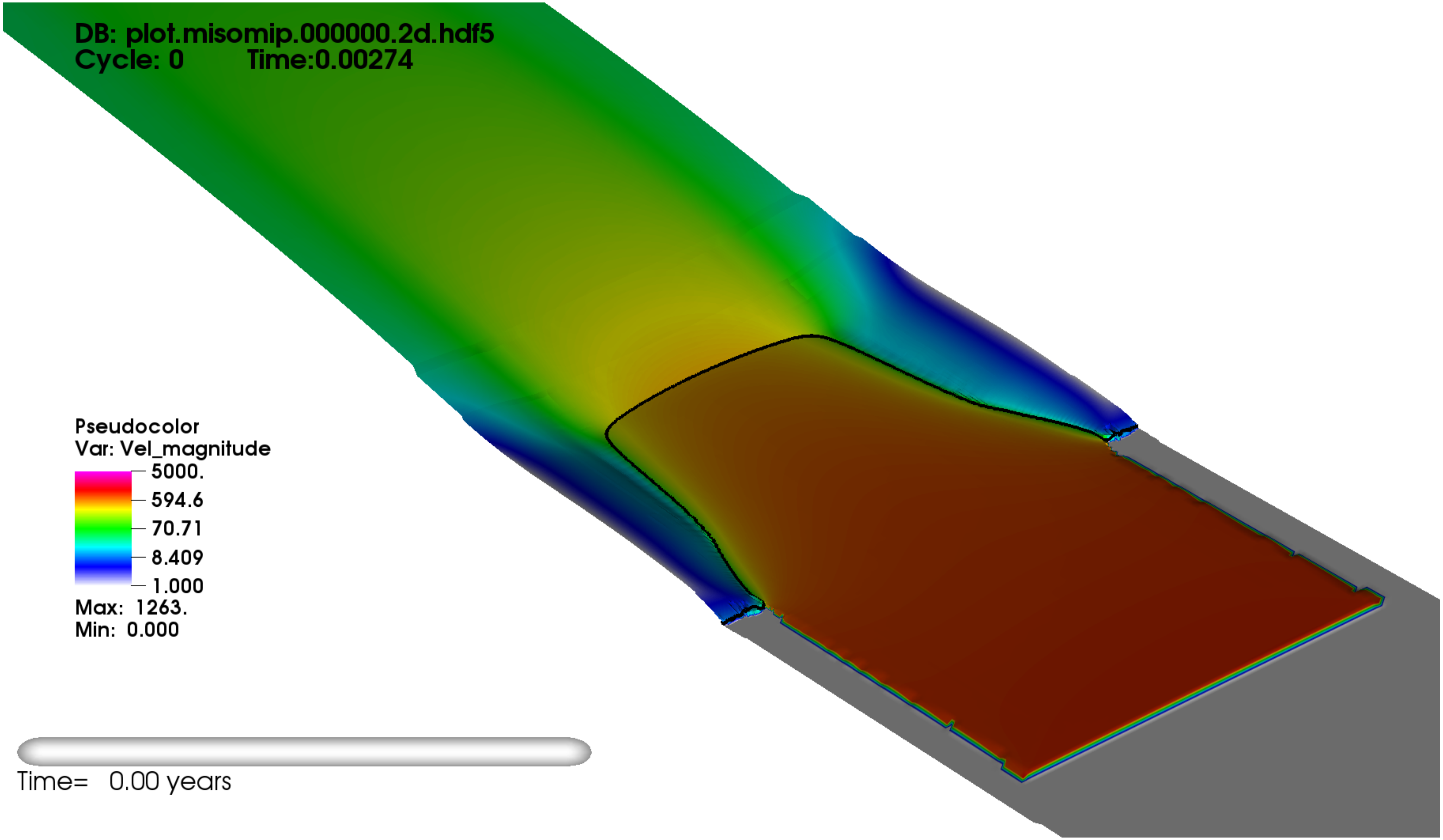
Steady-state initial condition



Fully-retreated condition

- Marine Ice Sheet-Ocean Model Intercomparison Project
- Ice Sheet coupled to Ocean Model through melt rates
- Driven by far-field forcing –
  - 0 < t < 100 years: Warm Phase (1 C)
  - 100 < t < 200 years: Cold Phase (-1.9 C)

MISOMIP



U.S. DEPARTMENT OF ENERGY | Office of Science

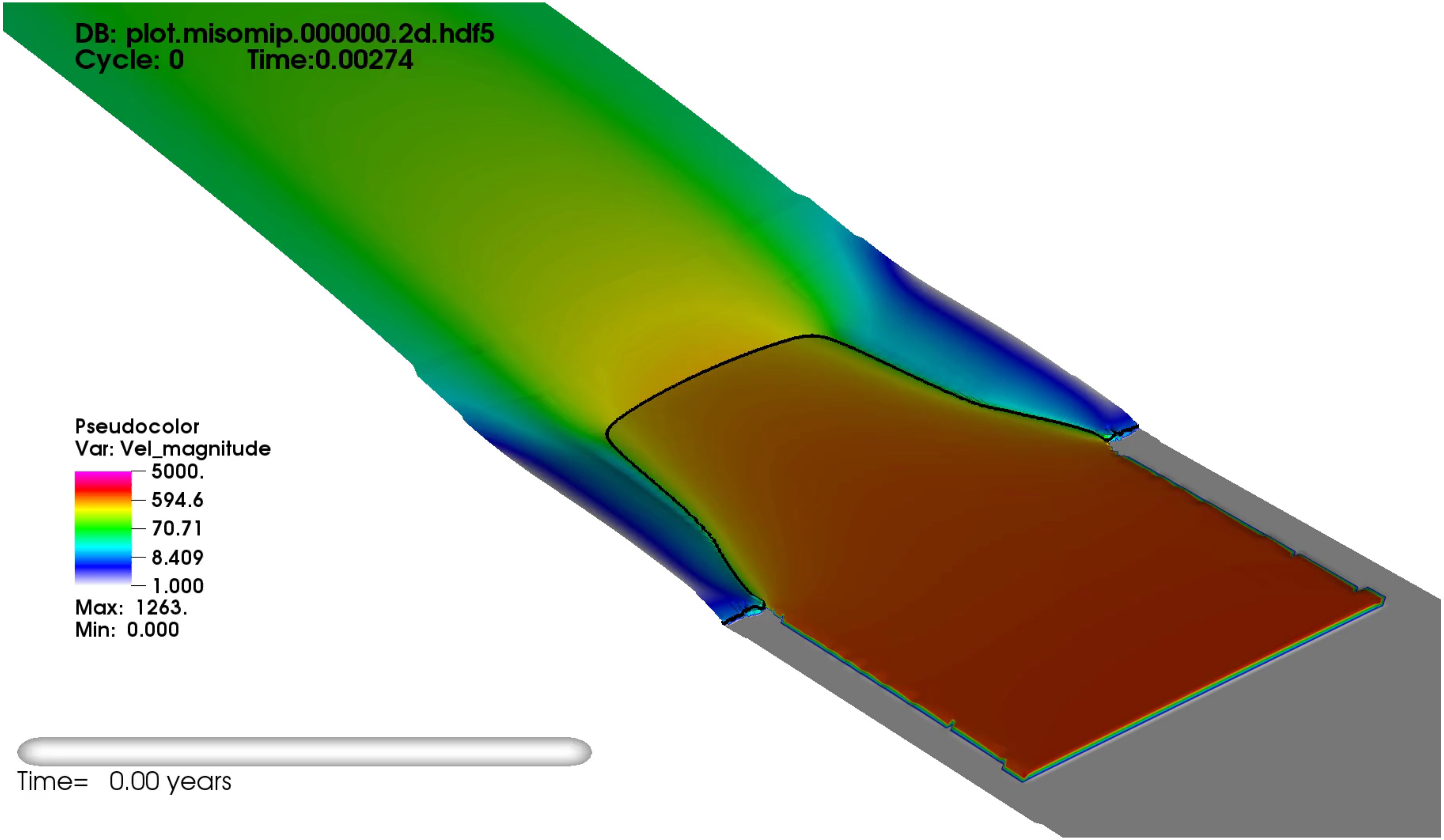
BISICLES

BERKELEY LAB

Los Alamos NATIONAL LABORATORY EST. 1943

University of BRISTOL

MISOMIP



U.S. DEPARTMENT OF ENERGY | Office of Science

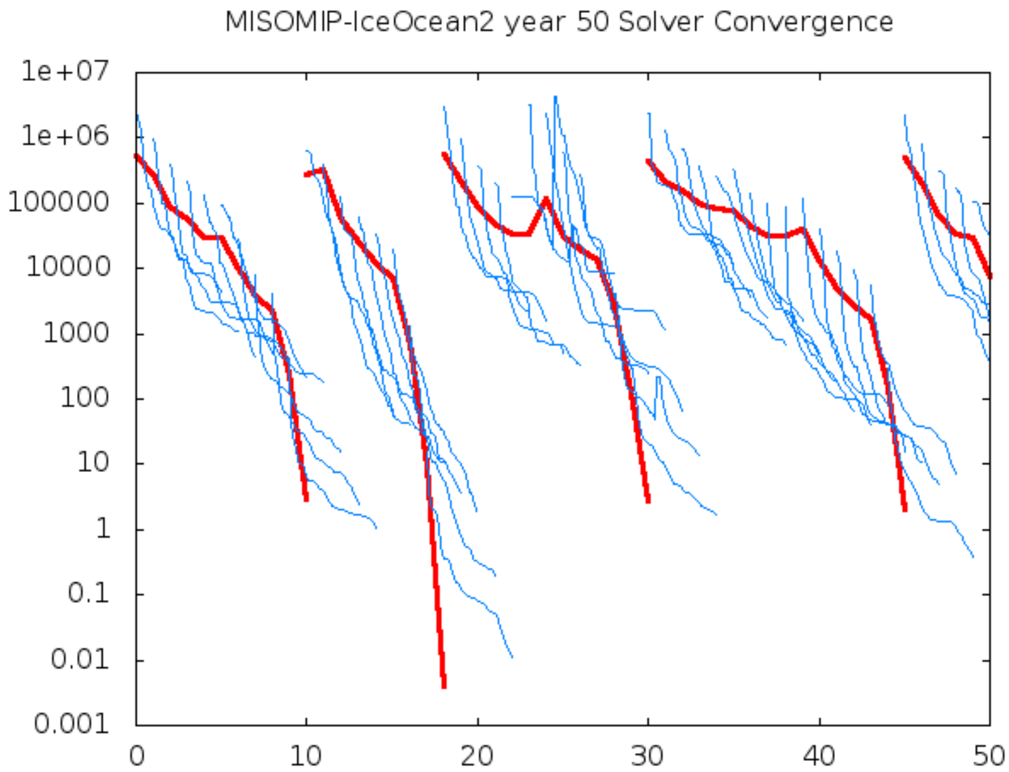
BISICLES

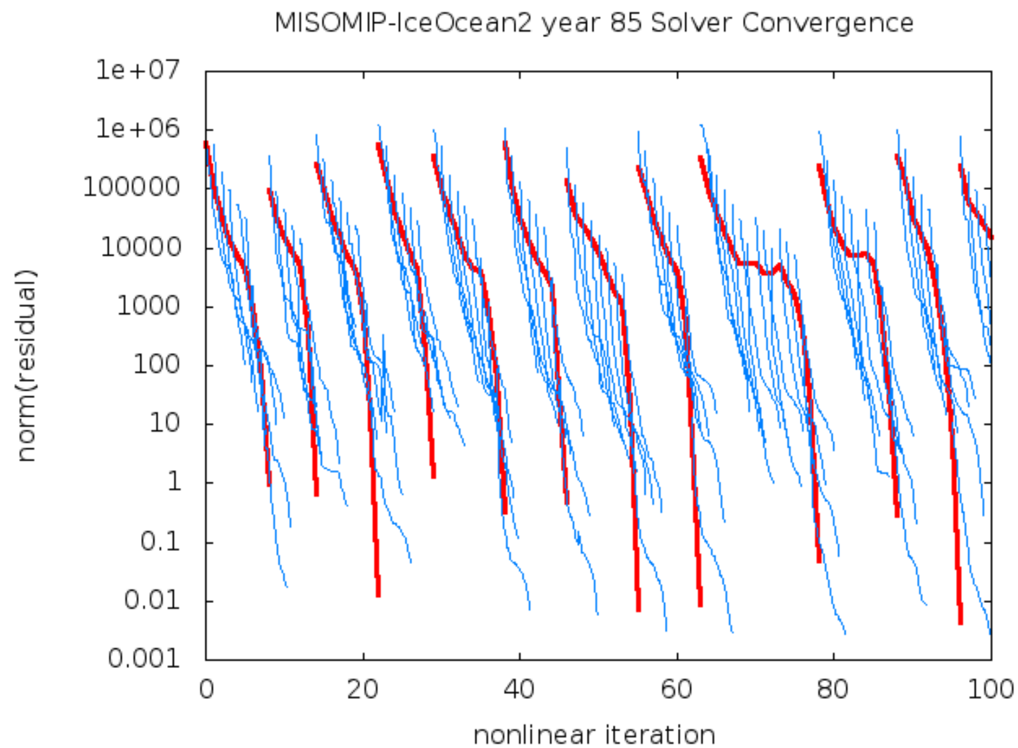
BERKELEY LAB

Los Alamos NATIONAL LABORATORY EST. 1943

University of BRISTOL

MISOMIP – Solver Convergence





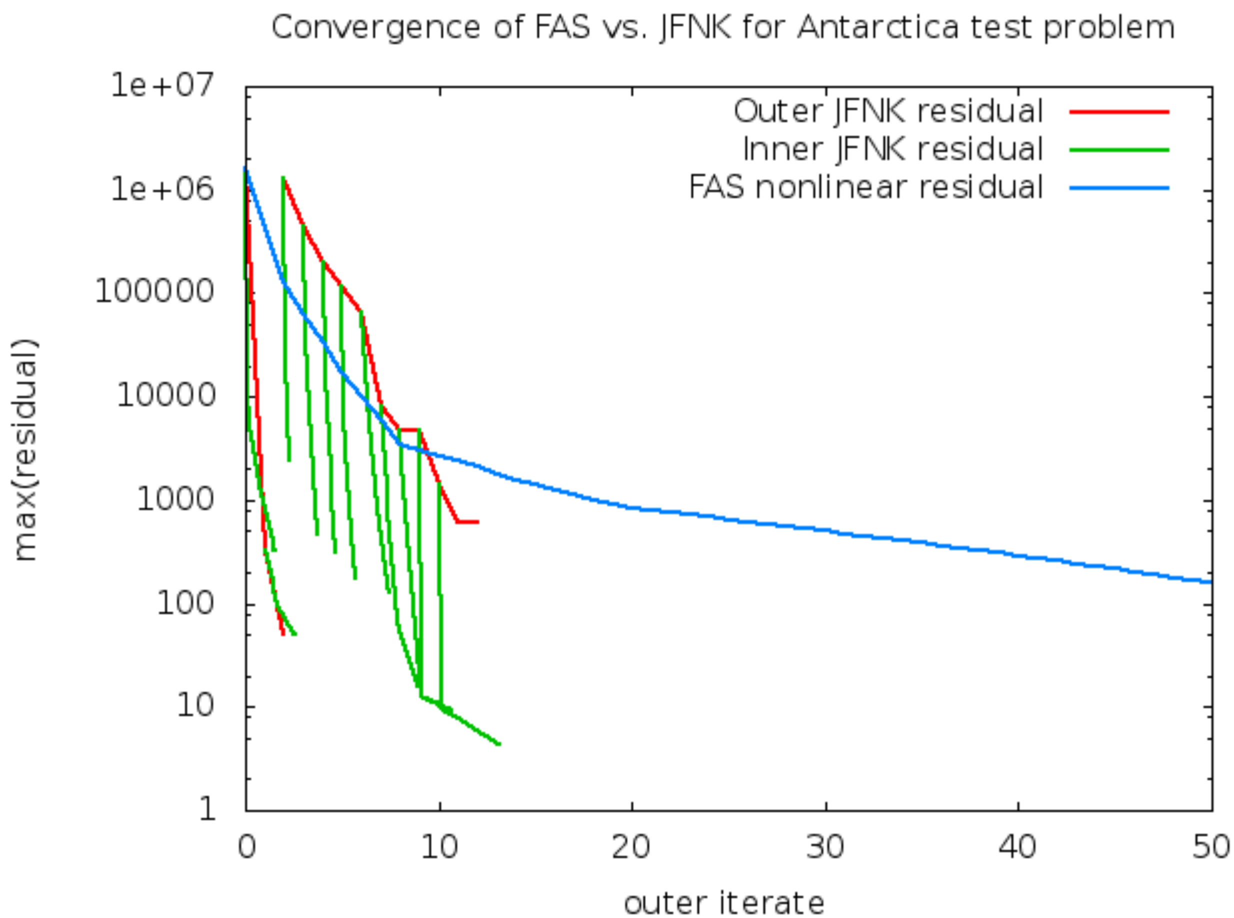
FAS Multigrid nonlinear solver

- Full Approximation Storage (FAS) – nonlinear multigrid

- Picard, JFNK:
  - linear solver nested inside of nonlinear one
  - Linear Multigrid solvers (residual-correction form) work well.

- FAS Multigrid – fully nonlinear solver (no outer solver)
  - Can outperform JFNK/MG
  - More robust (don't need good initial guess)
  - Simpler to implement and maintain
  - Nonlinear convergence similar to MG linear convergence

FAS Multigrid nonlinear solver (cont)



Solution time for 8 cores on linux desktop:
FAS-MG: 715 s   JFNK-MG: 1128 s

Acknowledgements:

- US Department of Energy Office of Science (ASCR/BER) SciDAC applications program (PISCEES)

- NERSC

U.S. DEPARTMENT OF ENERGY | Office of Science

BISICLES

BERKELEY LAB

Los Alamos NATIONAL LABORATORY EST. 1943

University of BRISTOL

Thank you!

Extras

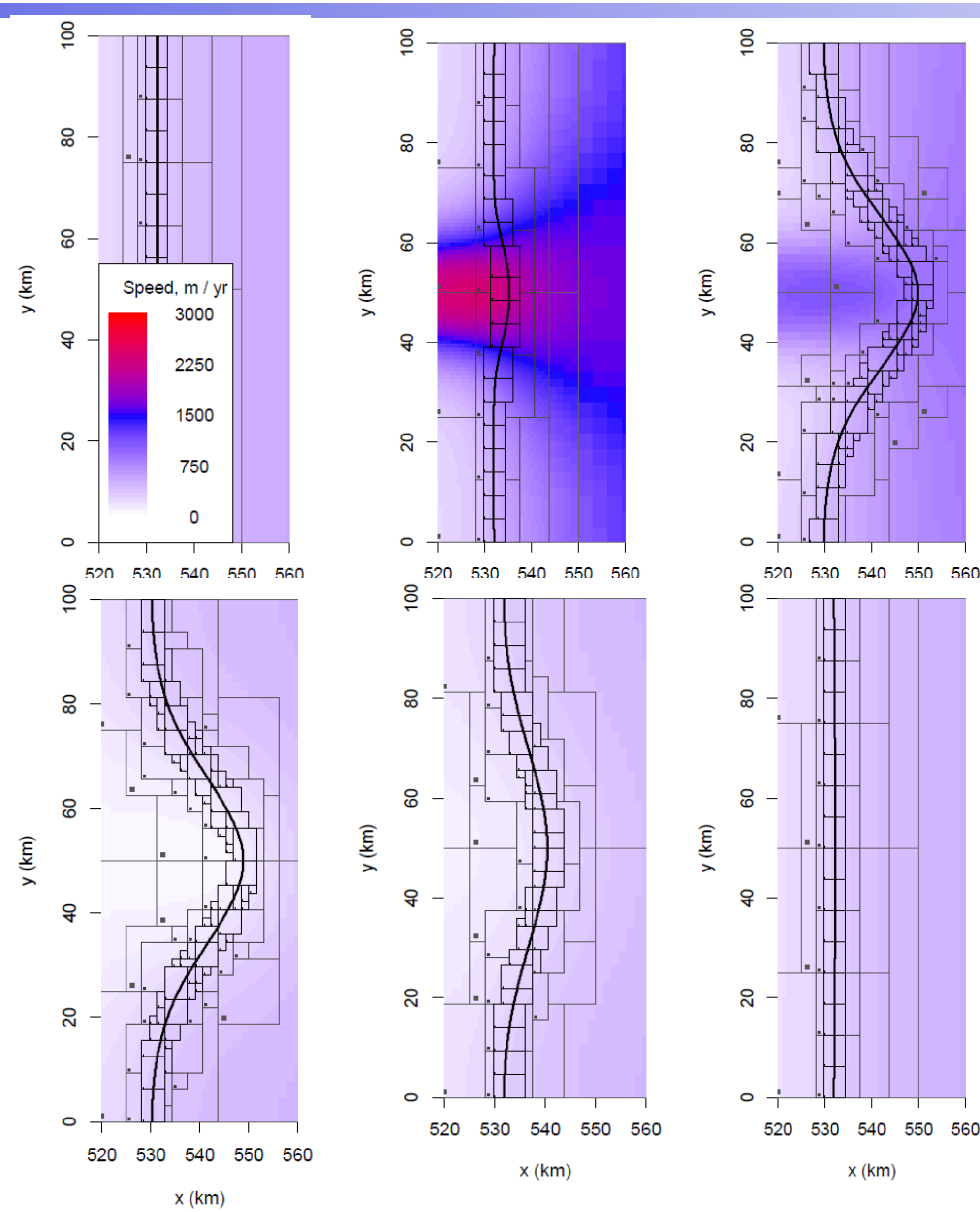
Grounding-line dynamics experiments

- Series of ice-sheet modeling community model intercomparison projects designed to understand issues in modeling of GLs
  - MISMIP, MISMIP3D, MISMIP+
- All point to a need for very fine spatial resolution to get GL dynamics right (sub-km in most cases)
- Prime use case for adaptive mesh refinement (AMR)

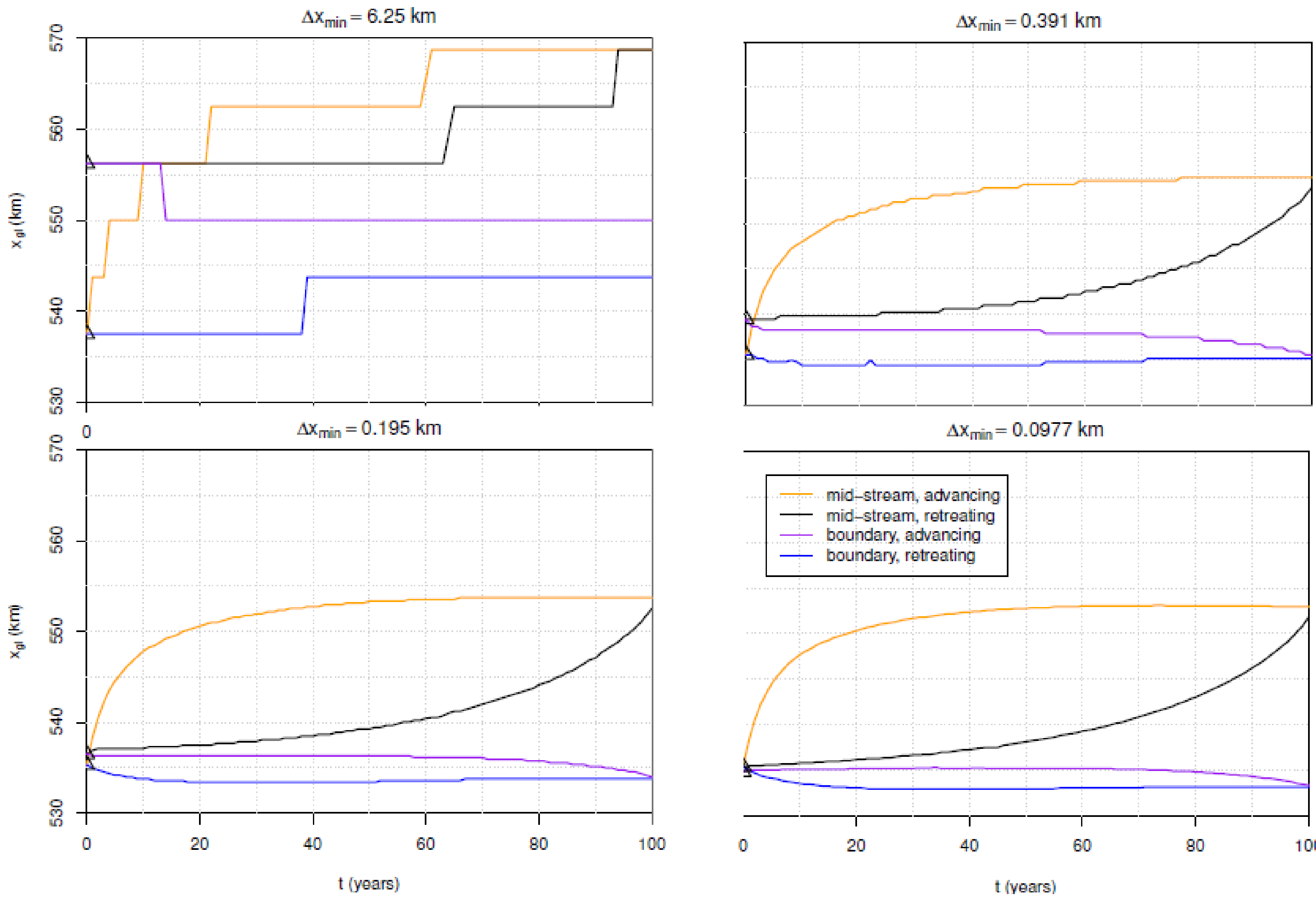
BISICLES Results - MISMIP3D

Experiment P75R:
(Pattyn et al (2011)

- Begin with steady-state (equilibrium) grounding line.
- Add Gaussian slippery spot perturbation at center of grounding line
- Ice velocity increases, GL advances.
- After 100 years, remove perturbation.
- Grounding line should return to original steady state.
- Figures show AMR calculation:
  - $\Delta x_0 = 6.5 km$ base mesh,
  - 5 levels of refinement
  - Finest mesh $\Delta x_4 = 0.195 km$.
  - t = 0, 1, 50, 101, 120, 200 *yr*
- Boxes show patches of refined mesh.
- GL positions match Elmer (full-Stokes)

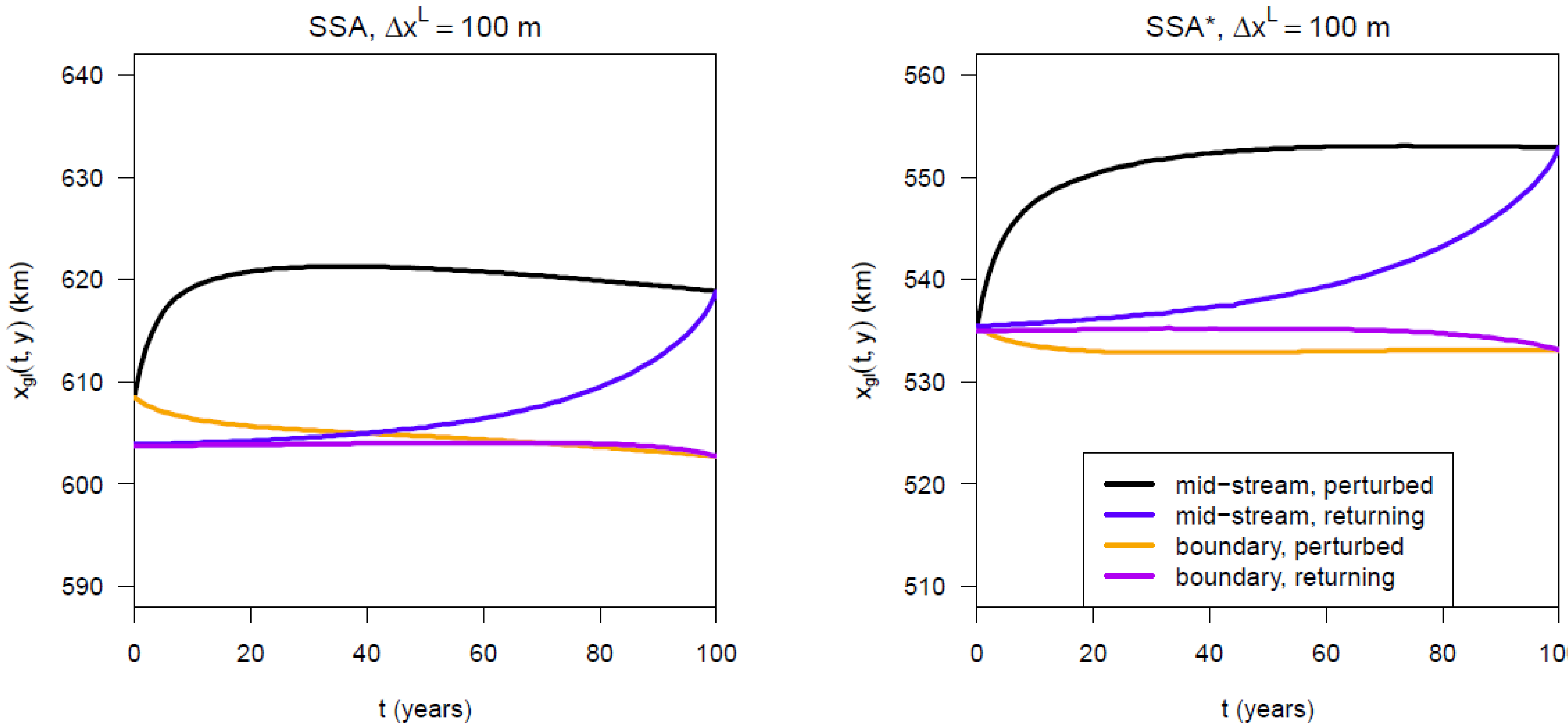


MISMIP3D (cont): L1L2 (SSA*) Spatial Resolution



- Very fine (~200 m) resolution needed to achieve reversability!

MISMIP3D: SSA vs. "L1L2" or "SSA"*



- Direct comparison of SSA vs. SSA*
  - (fully resolved spatially, same numerics, etc)
  - Note difference in steady-state GL positions

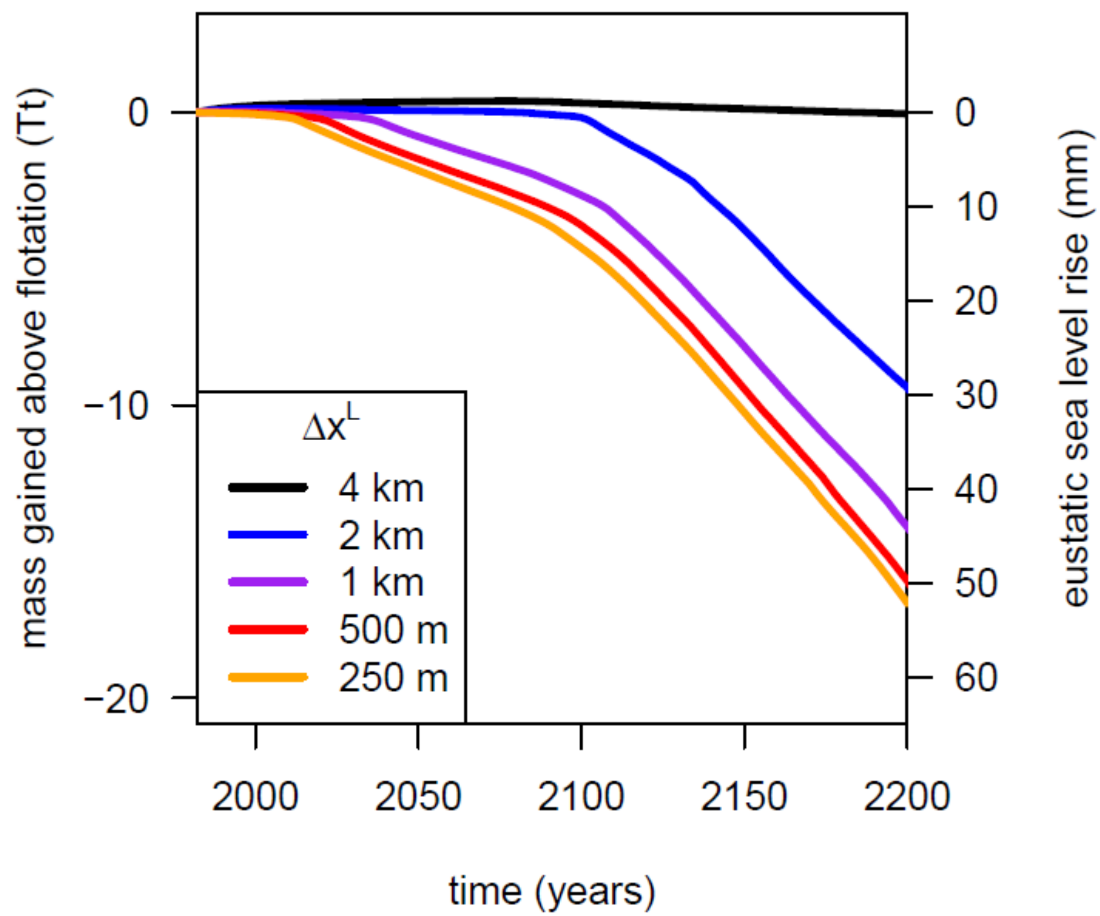
BISICLES Results – Ice2Sea Amundsen Sea

- Study of effects of warm-water incursion into Amundsen Sea.
- Results from Payne et al, (2012), submitted.
- Modified 1996 BEDMAP geometry (Le Brocq 2010), basal traction and damage coefficients to match Joughin 2010 velocity.
- Background SMB and basal melt rate chosen for initial equilibrium.
- SMB held fixed.
- Perturbations in the form of additional subshelf melting:
  - derived from FESOM circumpolar deep water
  - ~5 m/a in 21st Century,
  - ~25 m/a in 22nd Century.

Ice2Sea Amundsen (cont)

- Need at least 2 km resolution to get any measurable contribution to SLR.
- Appears to converge at first-order in Δx

SLR vs. year, Amundsen Sea Sector

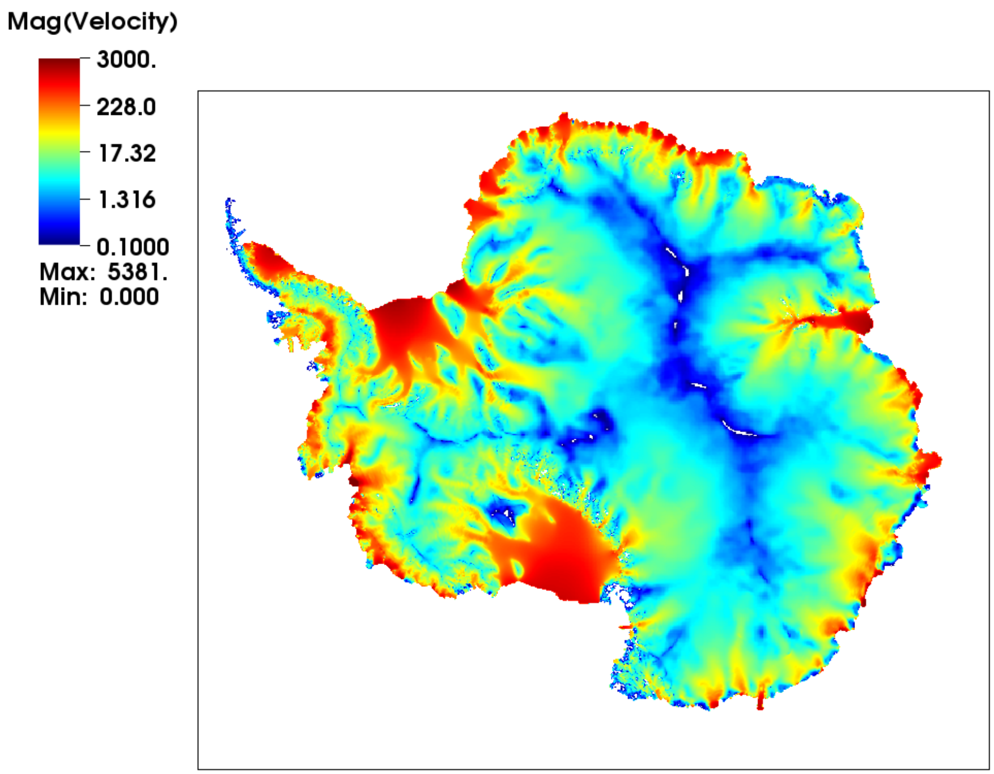


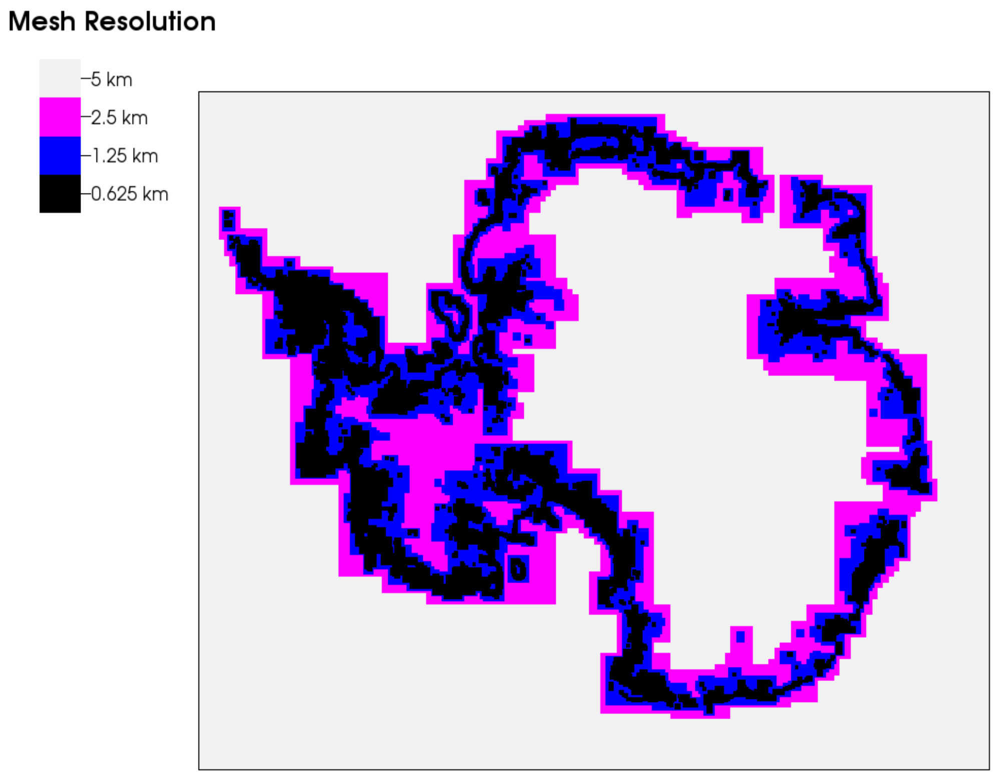
Ice2Sea Amundsen (cont) – Thwaites?

- In 400 year run, Thwaites destabilizes as well.
- Essentially same forcing as previous run, subshelf melting held constant past 2200.
  - More melting: extra 5 m/a
- Thwaites is very stable, ***until it tips***.

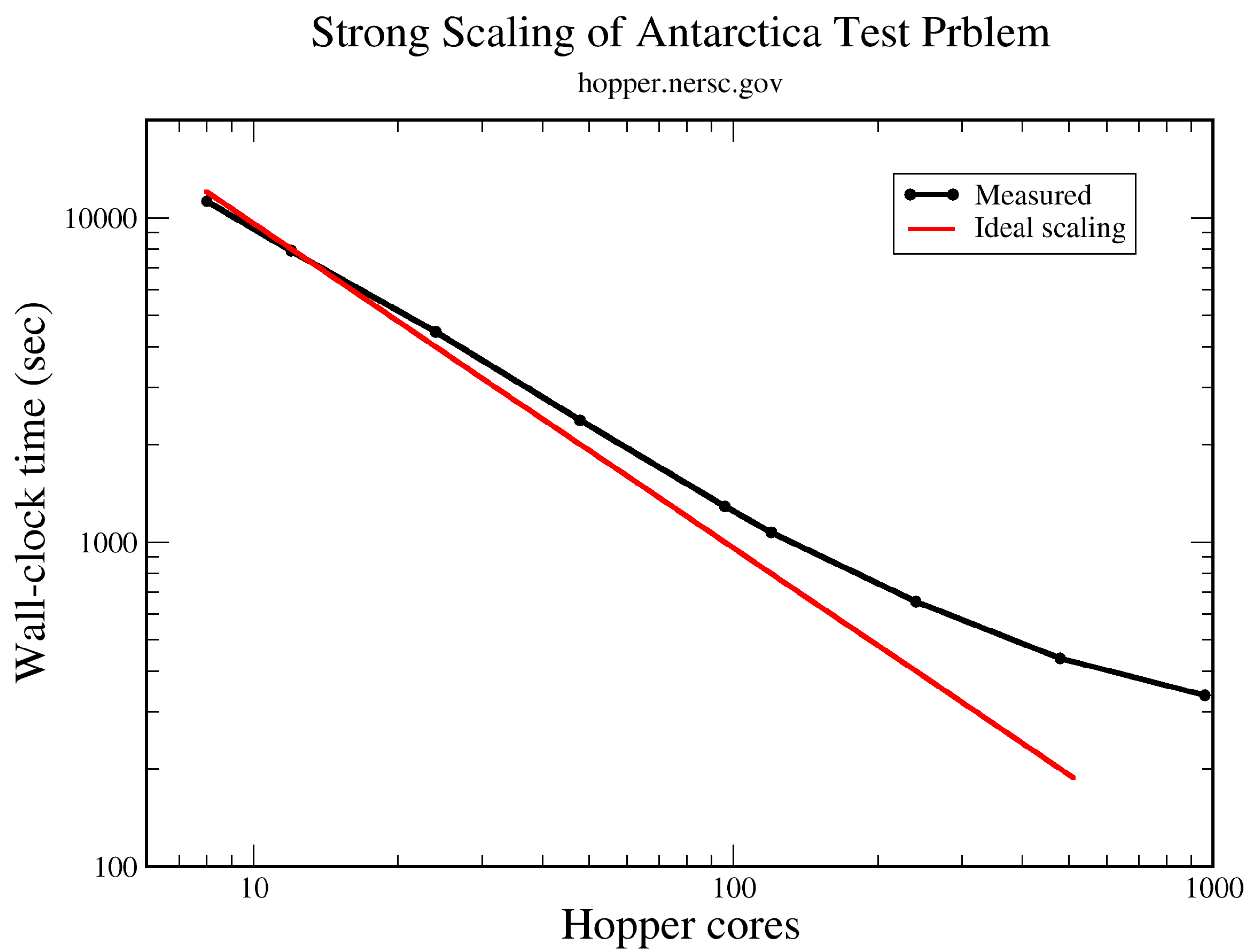
Antarctica (Ice2Sea)

- Refinement based on Laplacian(velocity), grounding lines
- 5 km base mesh with 3 levels of refinement
  - base level (5 km): 409,600 cells (100% of domain)
  - level 1 (2.5 km): 370,112 cells (22.5% of domain)
  - Level 2 (1.25 km): 955,072 cells (14.6% of domain)
  - Level 3 (625 m): 2,065,536 cells (7.88% of domain)



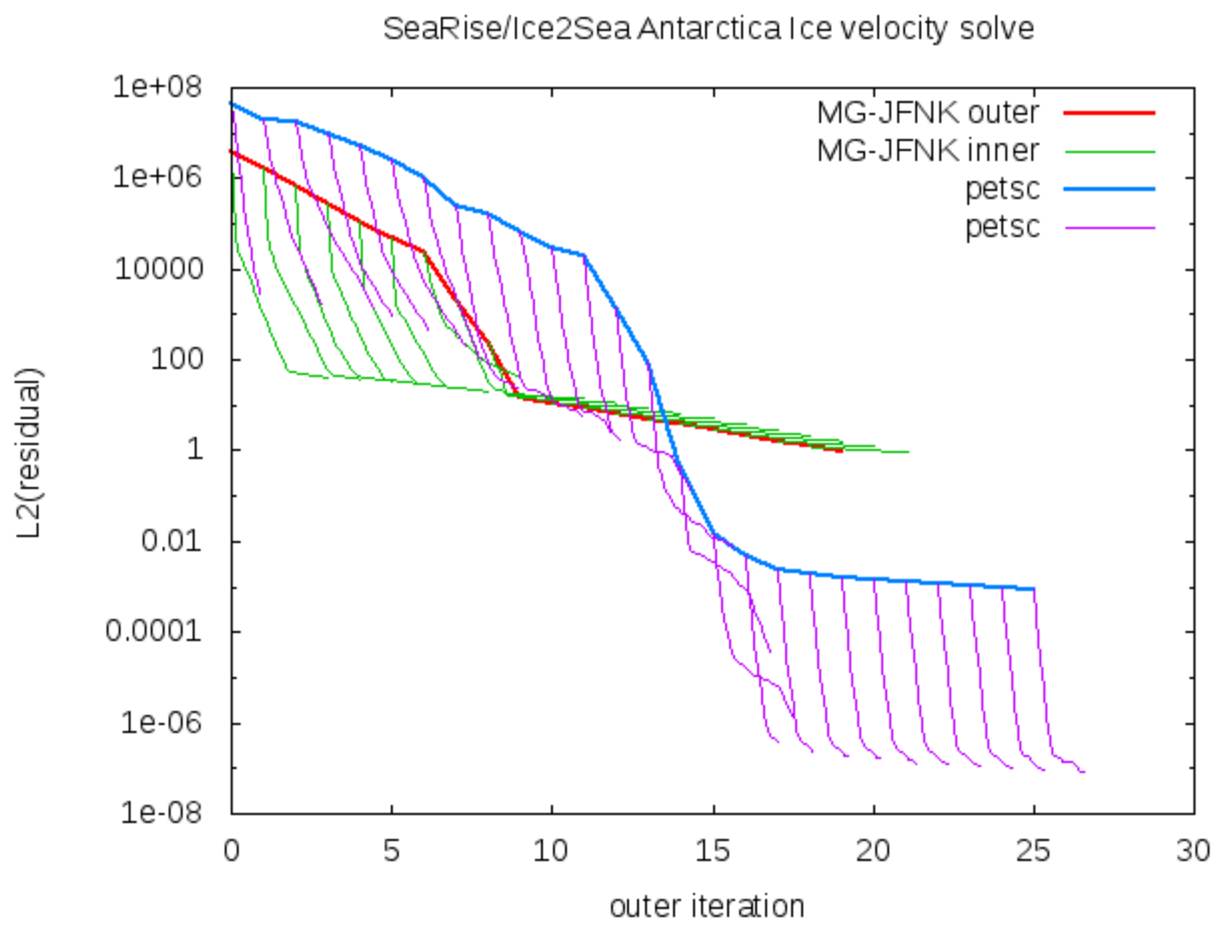


Parallel scaling, Antarctica benchmark



(Preliminary scaling result – includes I/O and serialized initialization)

Linear Solvers – GAMG vs. Geometric MG



Temporal Discretization

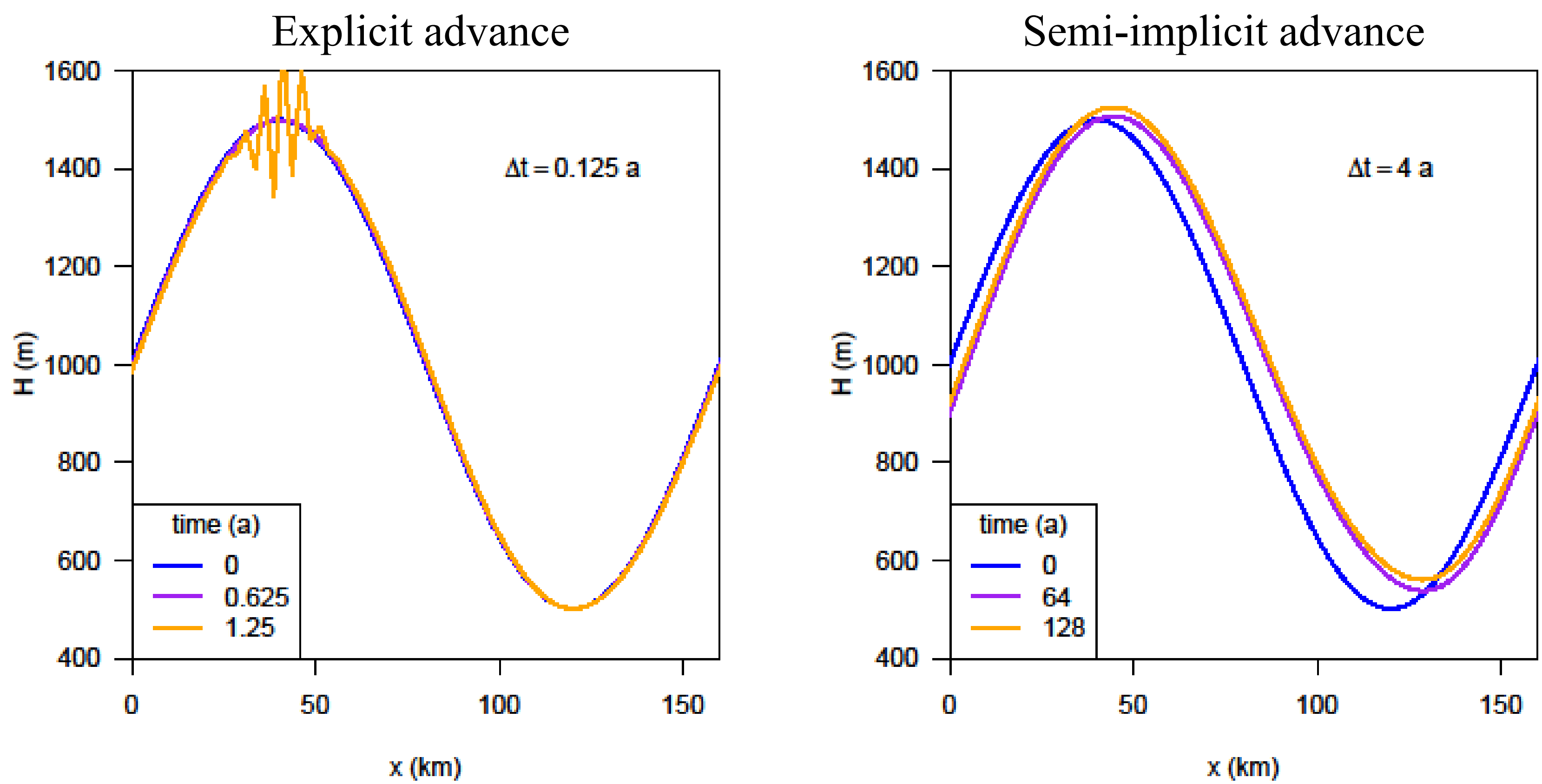
Update equation for H: $\partial H/\partial t + \nabla\cdot(u\,H) = S$

- "looks" like hyperbolic advection equation (explicit scheme, Courant stability -- $\Delta t \propto \Delta x$)
- Velocity field has ∇H piece – diffusion equation for H ($\Delta t \propto \Delta x^2$!)

- Strategy (Cornford) – try to factor out diffusive flux and discretize as an advection-diffusion equation:
  - $F = u\,H = F_{advective} + F_{diffusive}$
  - $F_{diffusive} = -D\,\nabla H$
  - Now solve: $\partial H/\partial t + \nabla\cdot F_{advective} = \nabla\cdot(D\,\nabla H) + S$
    - Advective fluxes: explicit update using unsplit 2nd Order PPM scheme
    - Diffusive fluxes: implicit update (Backward Euler for now)

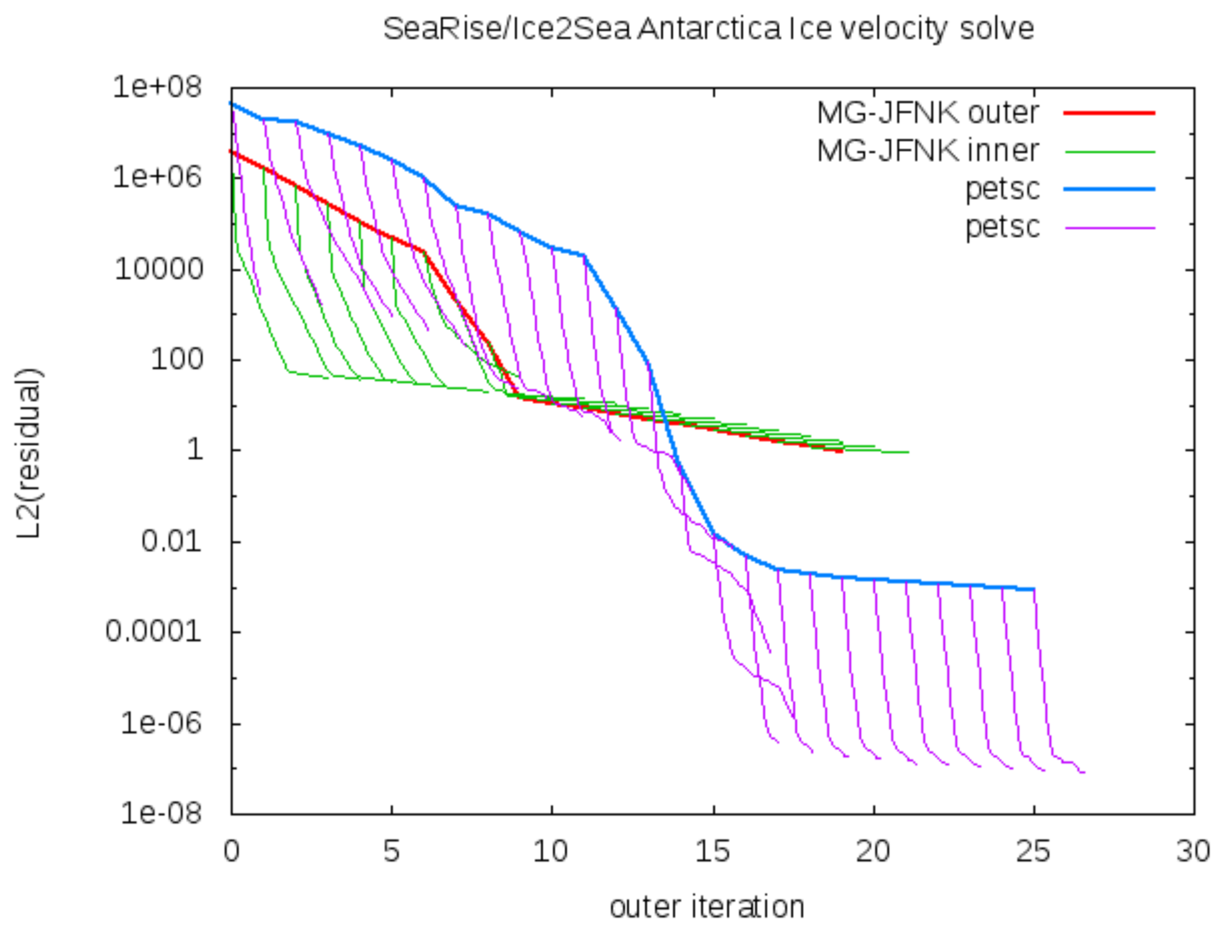
Temporal Discretization (cont)

- Test case based on ISMIP-HOM A geometry
- $\Delta x = 2.5\ km$, Δt_{CFL} = 5 a



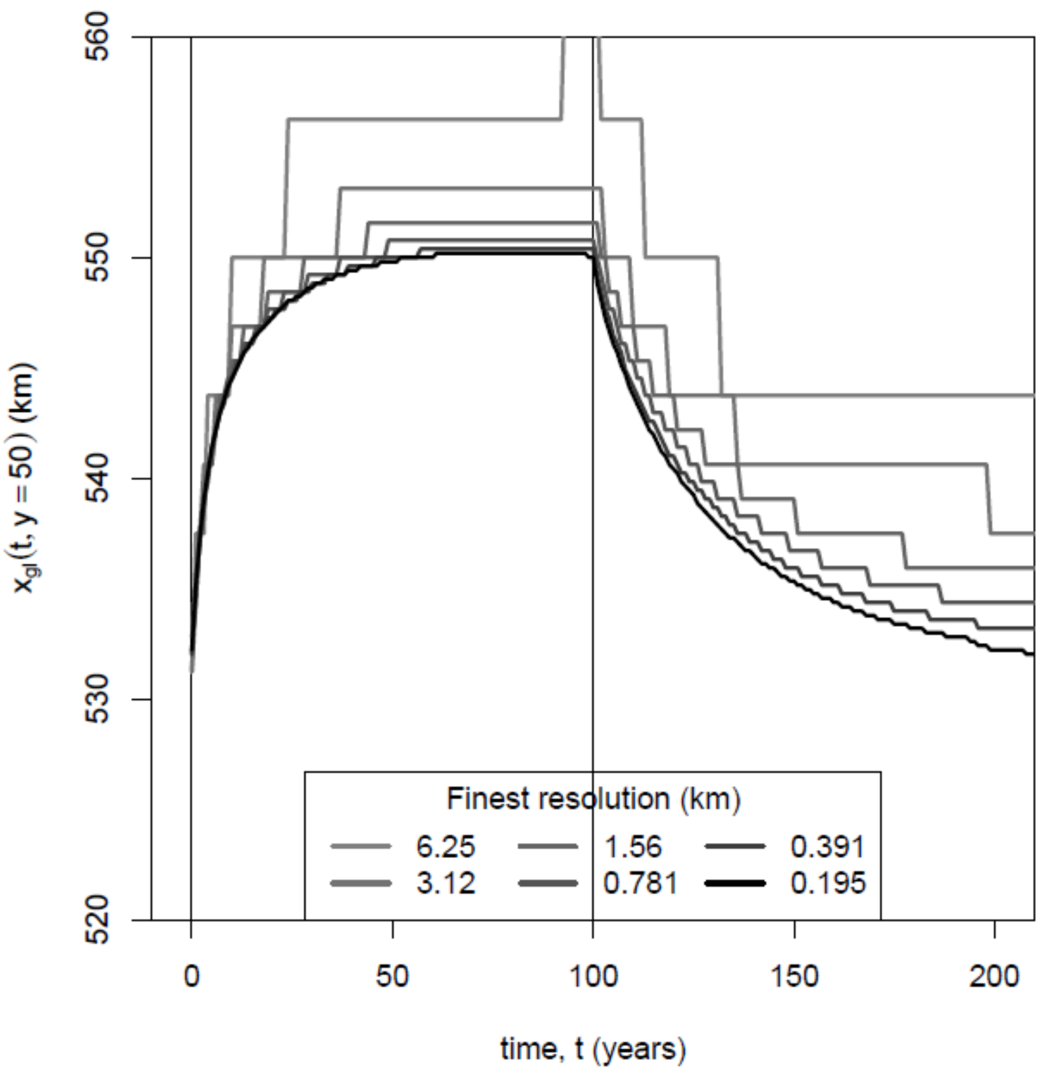
- Unfortunately, still run into stability issues finer than $\Delta x < 0.5\ km$

Linear Solvers – GAMG vs. Geometric MG



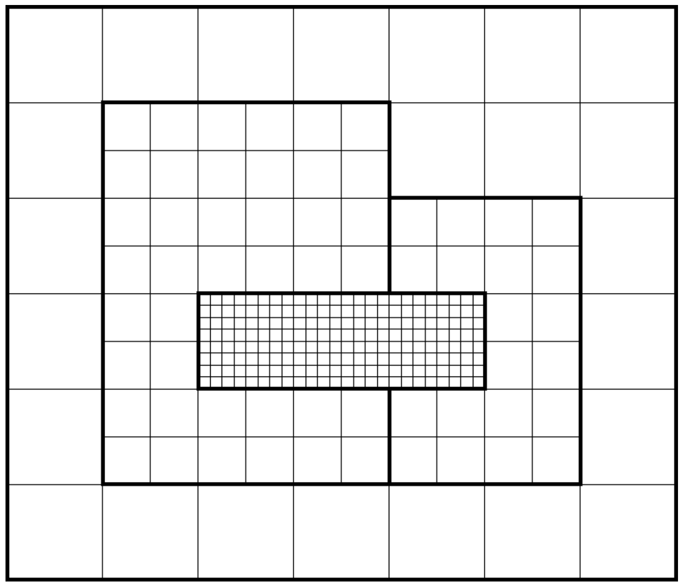
MISMIP3D: Mesh resolution

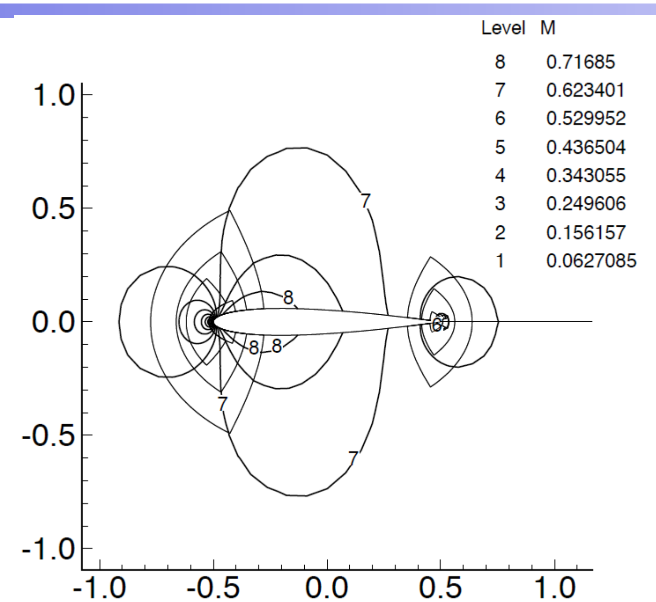
- Plot shows grounding line position x_{GL} at $y=50km$ *vs.* time for different spatial resolutions.
- $\Delta x = 0.195 km \rightarrow 6.25\ km$
- Appears to require finer than 1 km mesh to resolve dynamics
- Converges as O(Δx) (as expected)

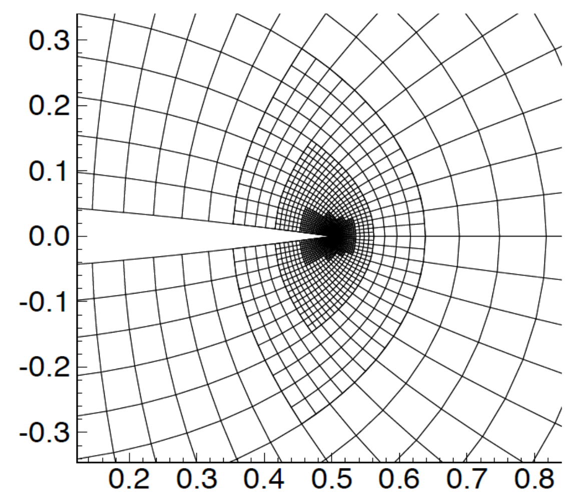


Block-Structured Local Refinement

- Refined regions are organized into rectangular patches.



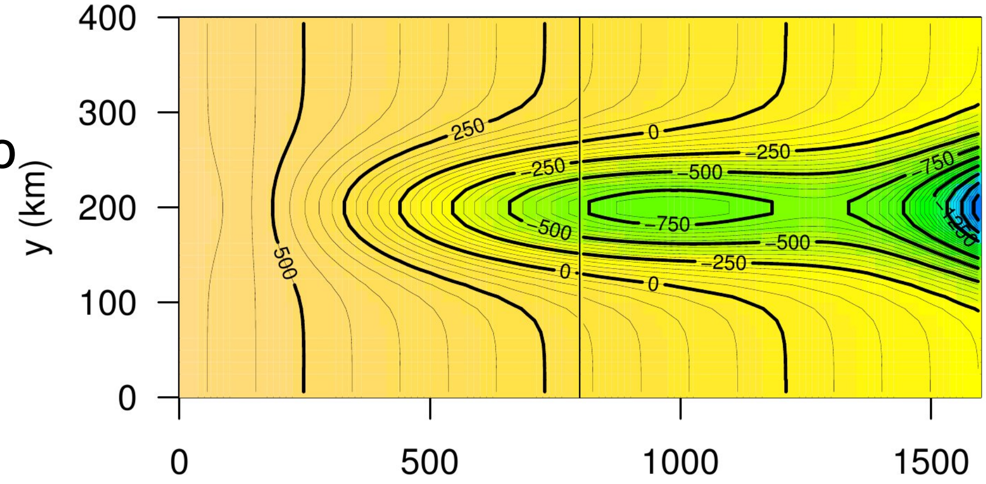


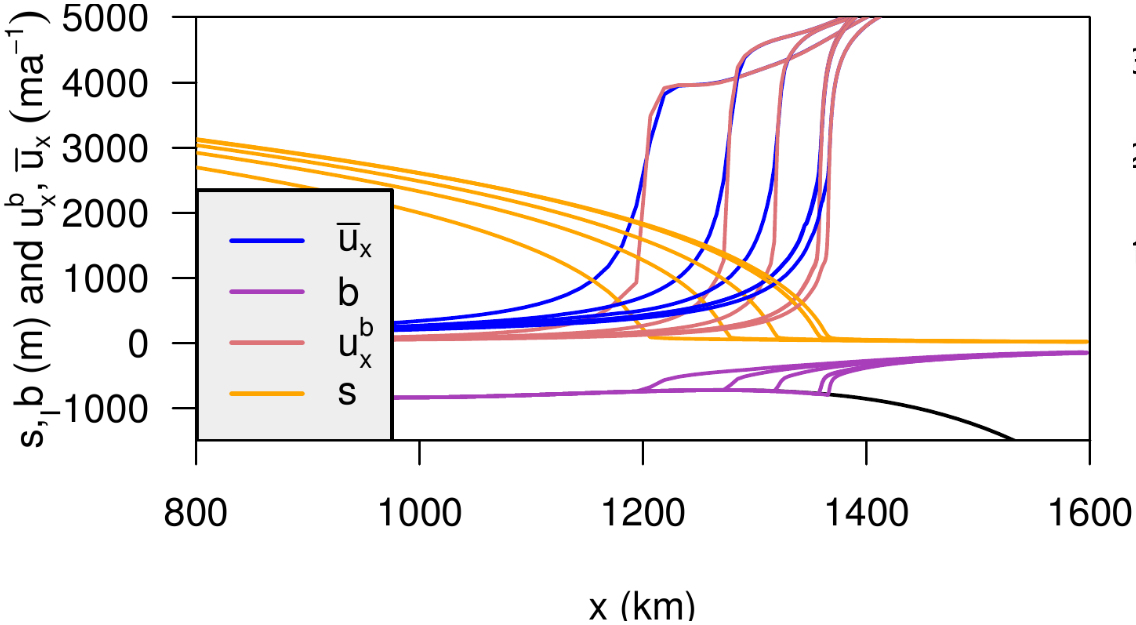


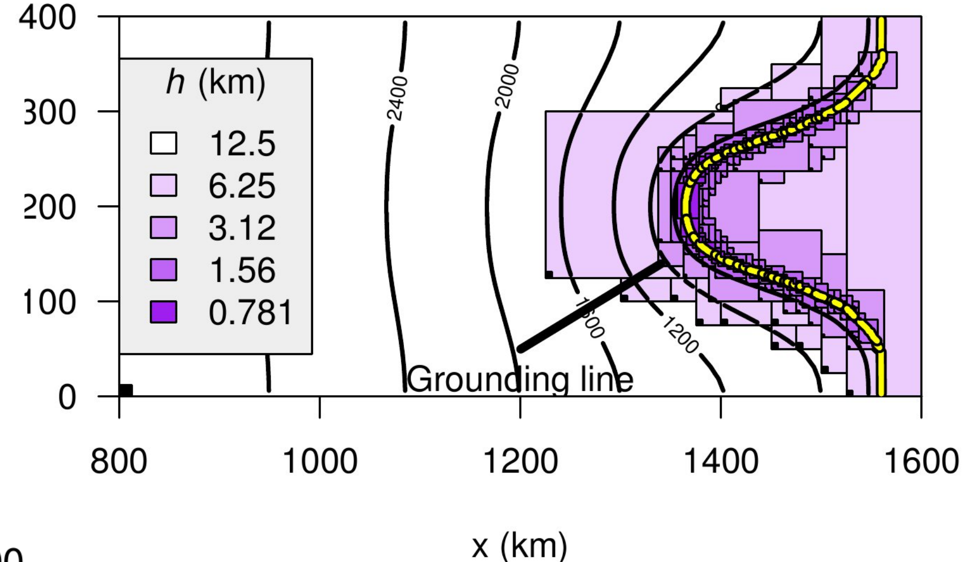
- *Algorithmic advantages:*
  - *Build on mature structured-grid discretization methods.*
  - *Low overhead due to irregular data structures, relative to single structured-grid algorithm.*

BISICLES results - Grounding line study

- Bedrock topography based on Katz and Worster (2010)
- Evolve initially uniform-thickness ice to steady state
- Repeatedly add refinement and evolve to steady state
- G.L. advances with finer resolution
- Appear to need better than 1 km

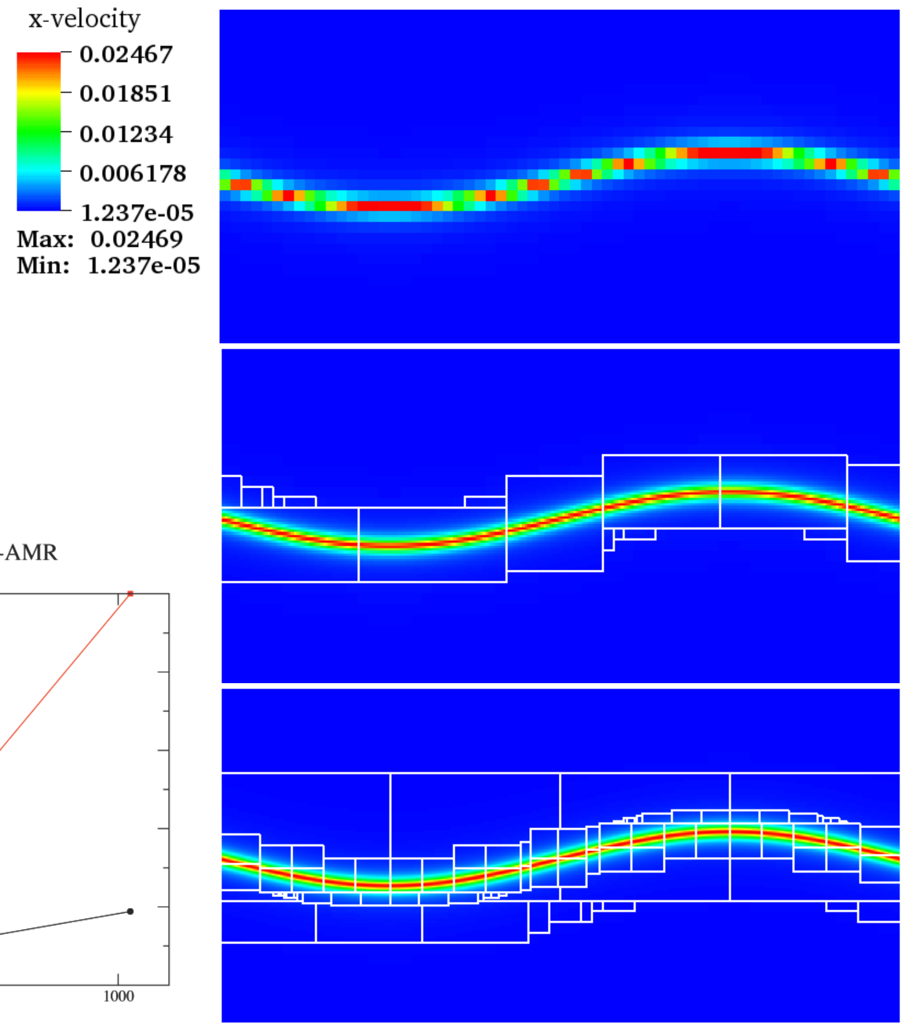


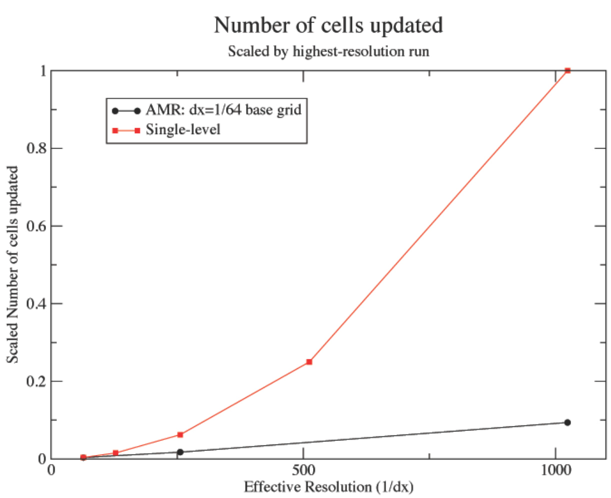


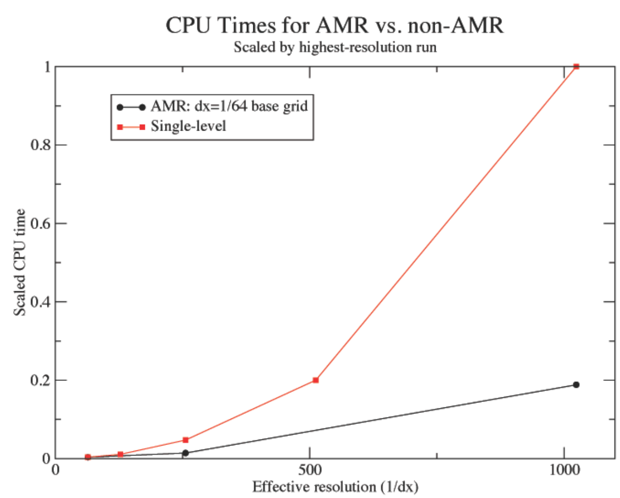


BISICLES Results

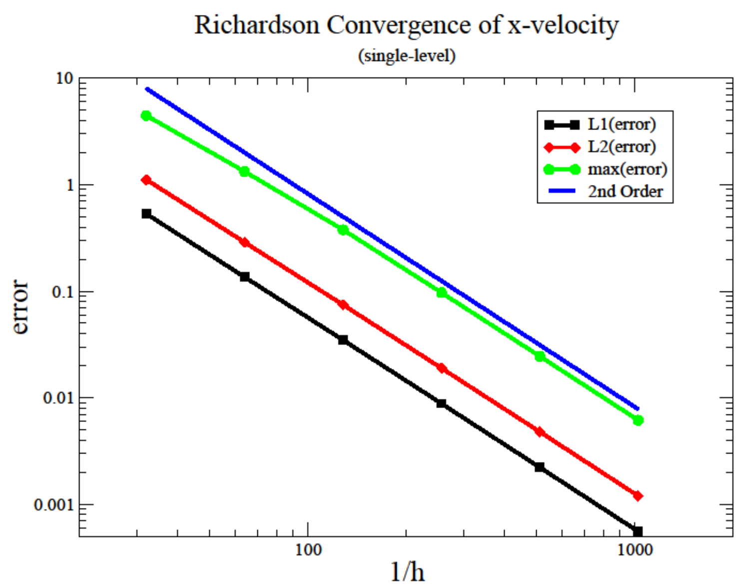
- Ice-stream Simulation
  [based on Pattyn et al (2008)]:
  - High resolution is required to accurately resolve the ice stream.
  - AMR simulation allows high resolution around the ice stream at a fraction of the cost of a uniformly refined mesh.

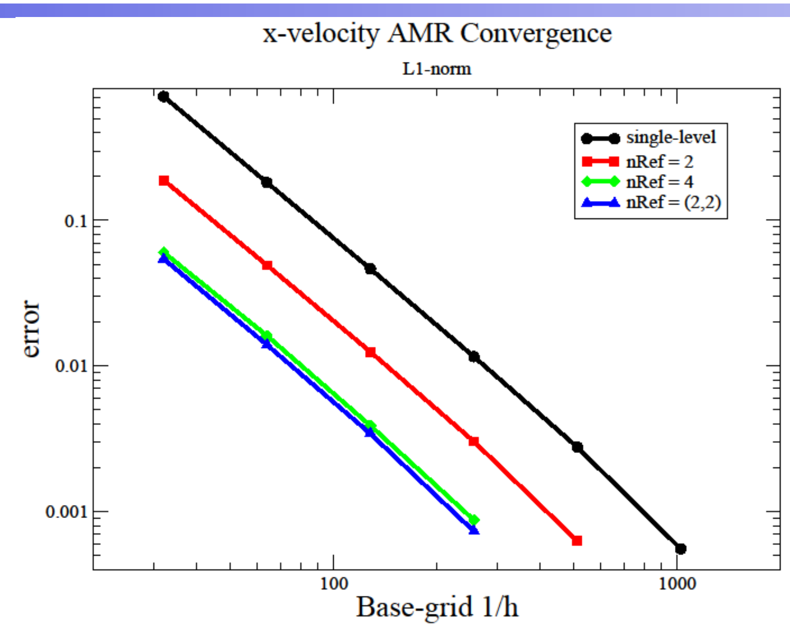


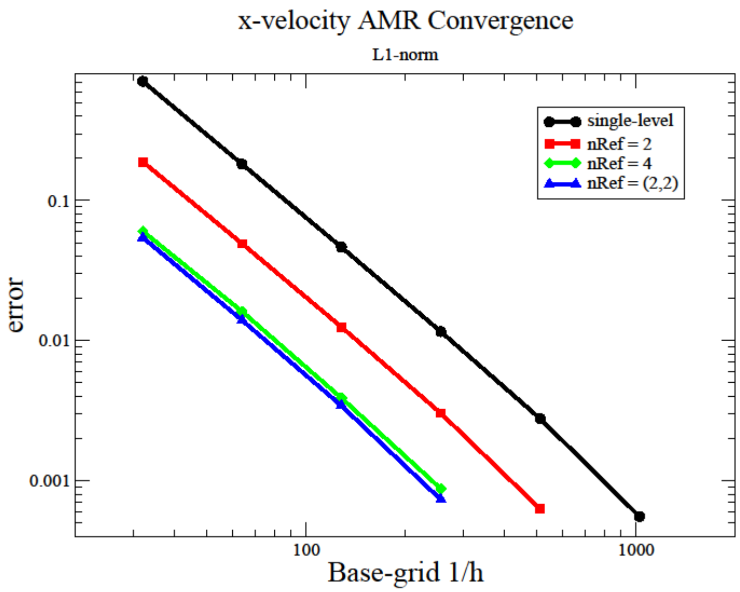


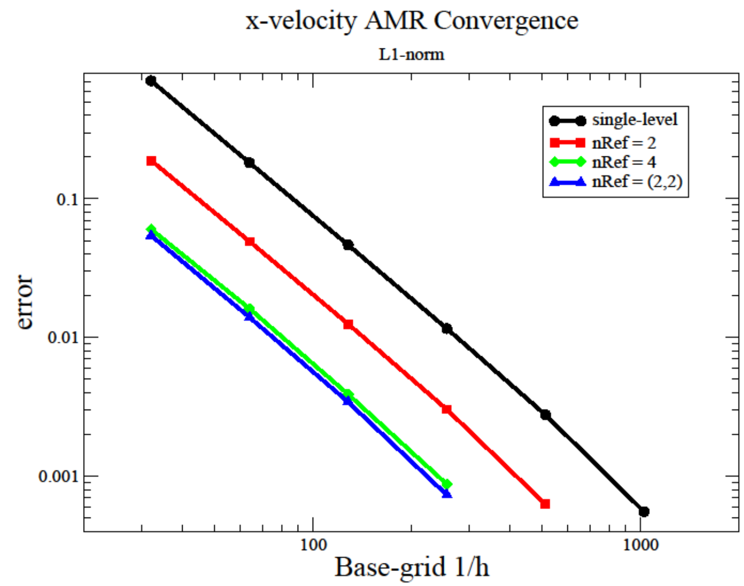


Numerical Accuracy and Convergence









MISMIP+

- "Child of MISMIP3D"
  - Examined GL response of models to a localized change in bed friction
  - Clarified resolution requirements for reversible GL dynamics
  - **Large variation in steady-state GL position among models**
  - Conclusions about dynamical results clouded by this difference
  - Said nothing about response to subshelf melt forcing (buttressing?)

- Specific details still under development
  - Steady-state with reduced variation between models
    - Steady-state on upward-sloping bed (buttressing) -- Gudmundsson (2012)
    - Narrow-ish channel (still under discussion)
  - Perturbation due to subshelf melt anomaly – GL retreat
  - Reversibility? (return timescale seems long)
  - Primary contact – Steph Cornford (Bristol)