FELIX: advances in modeling forward and inverse ice-sheet problems

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Ice Sheet Modeling

Main components of an ice model:

- **Ice flow equations** (momentum and mass balance)

$$\begin{cases} -\nabla \cdot \sigma = \rho \mathbf{g} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

inertial terms neglected

- **Model for the evolution of the boundaries**
(thickness evolution equation)

$$\frac{\partial H}{\partial t} = H_{flux} - \nabla \cdot \int_z \mathbf{u}\, dz$$

- **Temperature equation**

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) - \rho c \mathbf{u} \cdot \nabla T + 2\dot{\varepsilon}\sigma$$

- **Coupling with other climate components (e.g. ocean, atmosphere)**

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Stokes Approximations

"Reference" model: **STOKES**[1]

$O(\delta^2)$ ***FO,*** Blatter-Pattyn first order model[2] (3D PDE, in horizontal velocities)

$O(\delta)$ Zeroth order, depth integrated models:
SIA, Shallow Ice Approximation (slow sliding regimes) ,
SSA Shallow Shelf Approximation (2D PDE) (fast sliding regimes)

$\simeq O(\delta^2)$ Higher order, depth integrated (2D) models: ***L1L2***[3], (L1L1)...

[1] *Gagliardini and Zwinger, 2008. The Cryosphere.*

[2] *Dukowicz, Price and Lipscomb, 2010. J. Glaciol.*

[3] *Schoof and Hindmarsh, 2010. Q. J. Mech. Appl. Math.*

Felix overview

- **Felix**[1] (Finite Element Land Ice eXperiments) is a C/C++ finite element implementation[2] of land ice models. It relies on Trilinos for data structure, for the solution of linear/nonlinear solvers and for adjoint/UQ capabilities.

- Models currently implemented are SIA, SSA, L1L2 and FO, which have been tested[3] against Ismip-Hom experiments and CISM simulations.

- The nonlinear system are solved using Newton method with exact Jacobian + continuation of regularization parameters to increase robustness.

- It is interfaced with the land ice modulus of MPAS (climate library, implements ocean and atmosphere models). Realistic simulation has been for ice2sea projects.

- Even if adjoint and UQ capabilities are in early development, Felix can leverage on several trilinos packages which introduce great flexibility. Among these we have:
  - Dakota, MOOCHO (Optimization / UQ)
  - Sacado (Automatic Differentiation)

[1] *Software currently developed under the DOE project PISCEES*
[2] *www.trilinos.sandia.org (albany), www.lifev.org*
[3] *Perego, Gunzburger, Burkardt,* Journal of Glaciology, 2012

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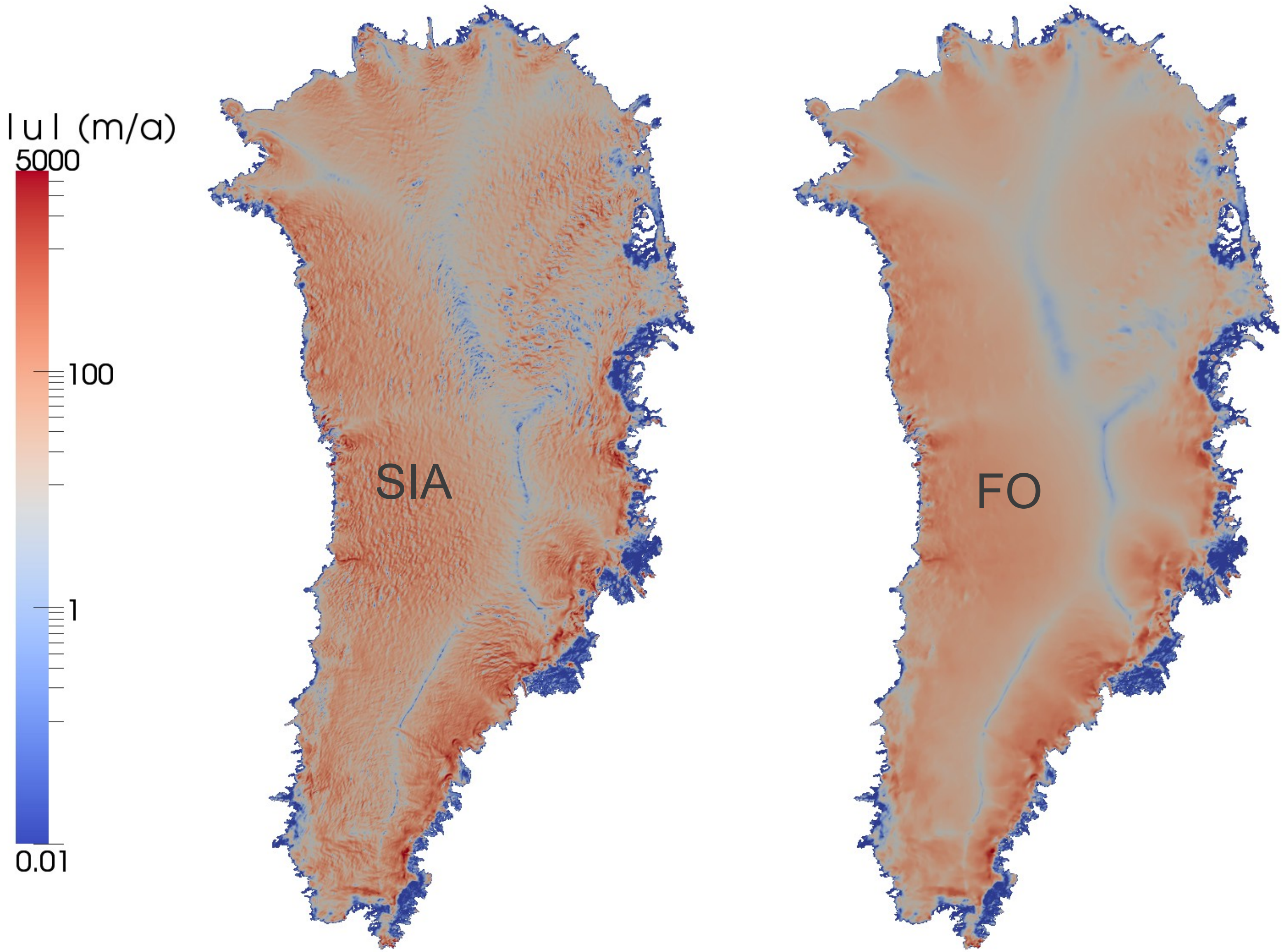
Outline

- Felix's features for solving Forward problems
  - *scalability of solvers*
  - *nonlinear solver options*
  - *coupling with the climate library MPAS*
  - *ice2sea runs*

- Preliminary work on Inverse/UQ problems
  - *UQ study on synthetic problems*
  - *Adjoint-based inversion for initial state*

Greenland steady state “benchmark”

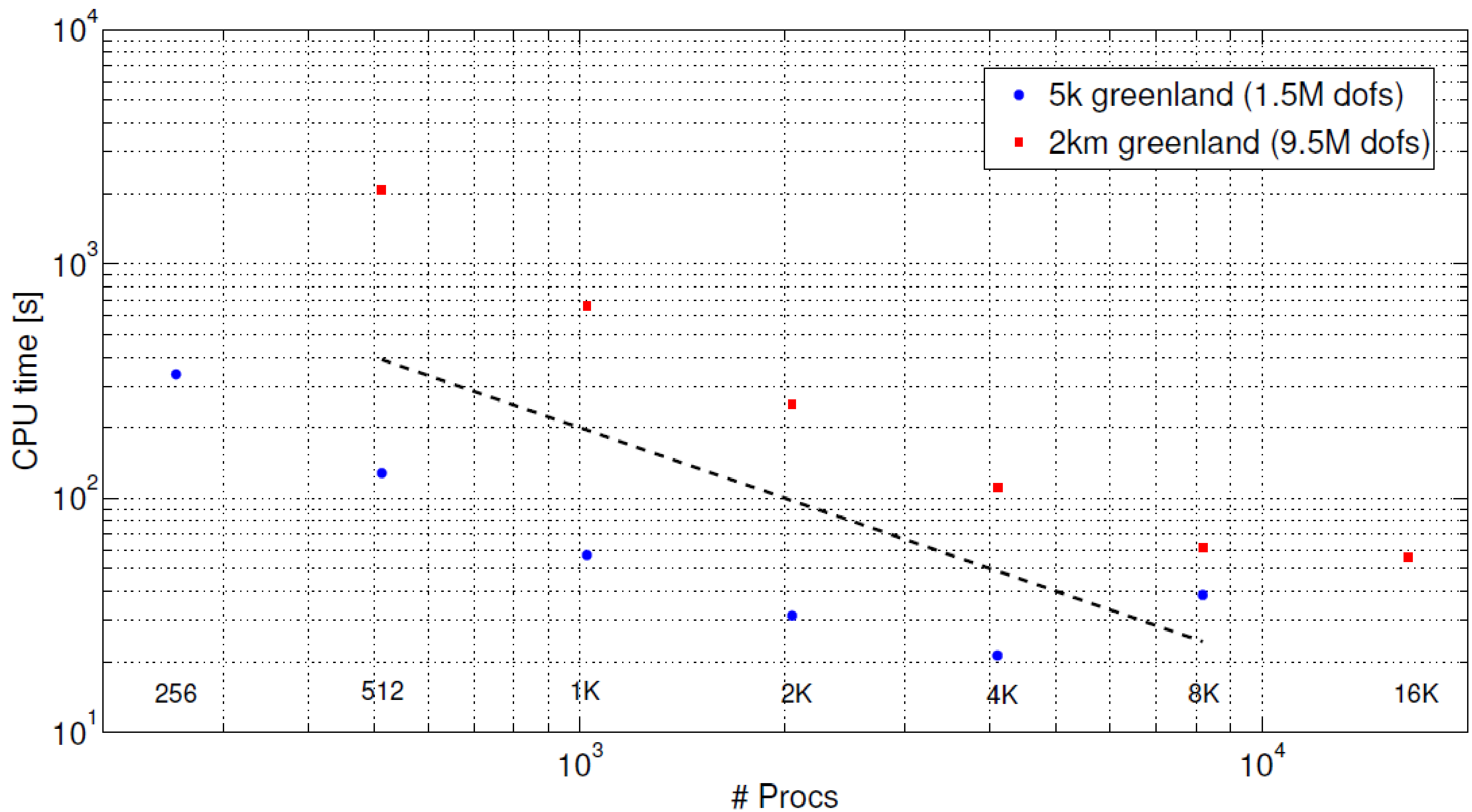
Greenland, 2km resolution, no-slip
FO steady problem used for scalability and convergence tests



Greenlad data: courtesy of J. Bamber and J. Griggs (Univ. of Bristol) as part of the Ice2Sea project.

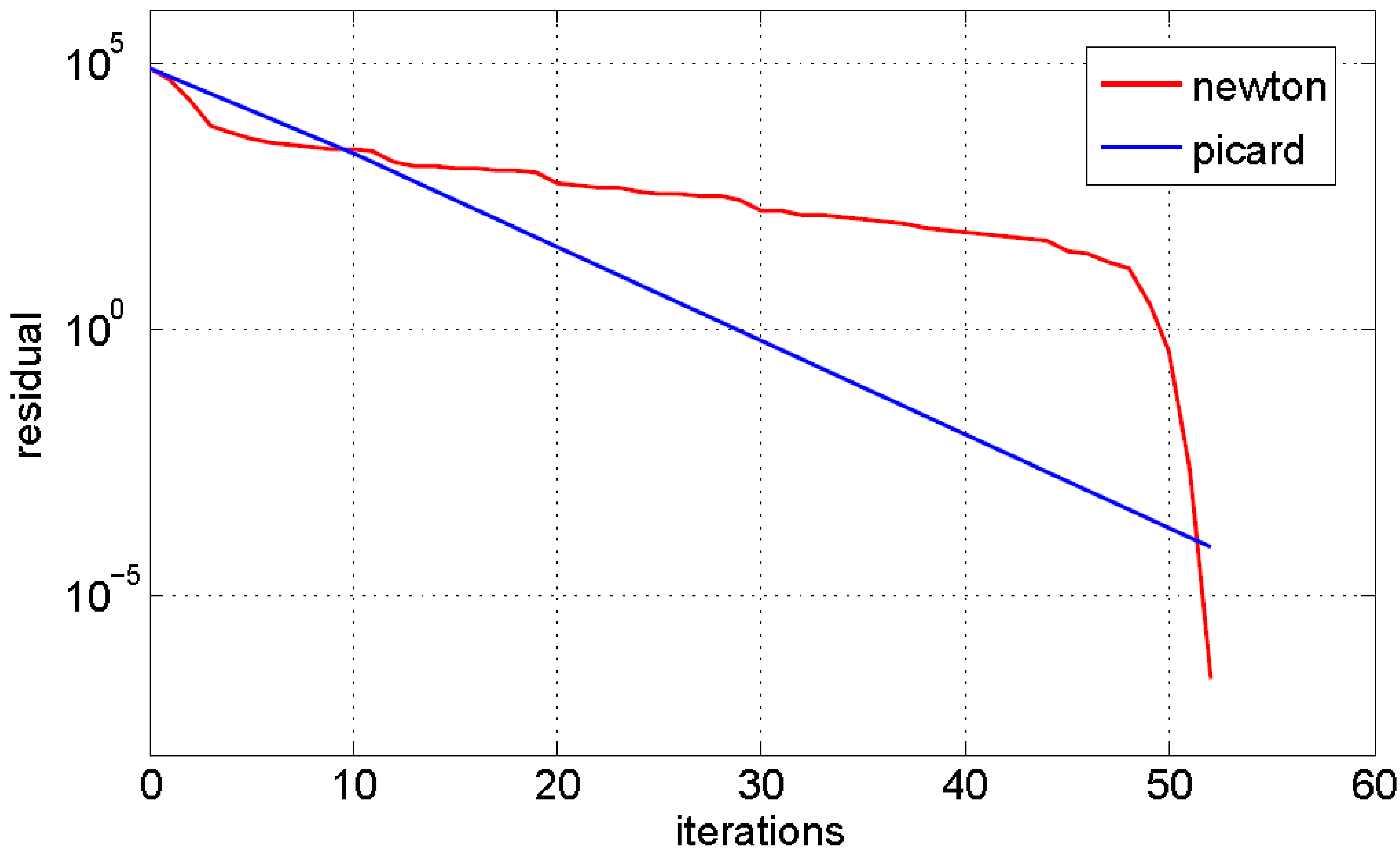
Greenland, FO: scalability results

Nonlinear Solver: Newton with exact Jacobian (NOX)
Linear Solver: CG/Gmres (BELOS)
Preconditioner: Additive Schwarz (IFPACK) with direct solver KLU on each subdomain.



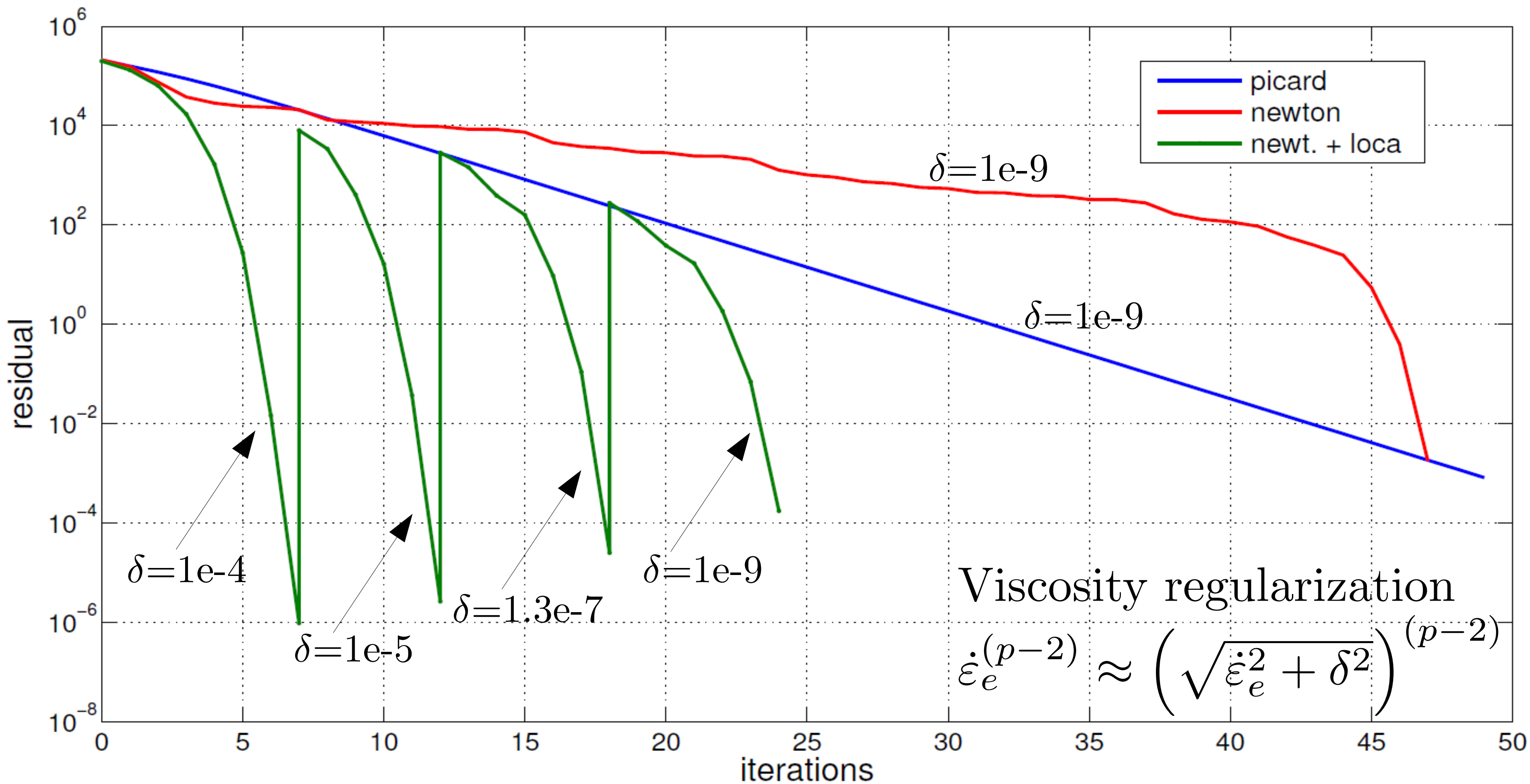
CPU time for the first time step.

Non linear solvers
Newton/Picard on Greenland



Non linear solvers
Increased Robustness with LOCA continuation method

M. P., A. Salinger

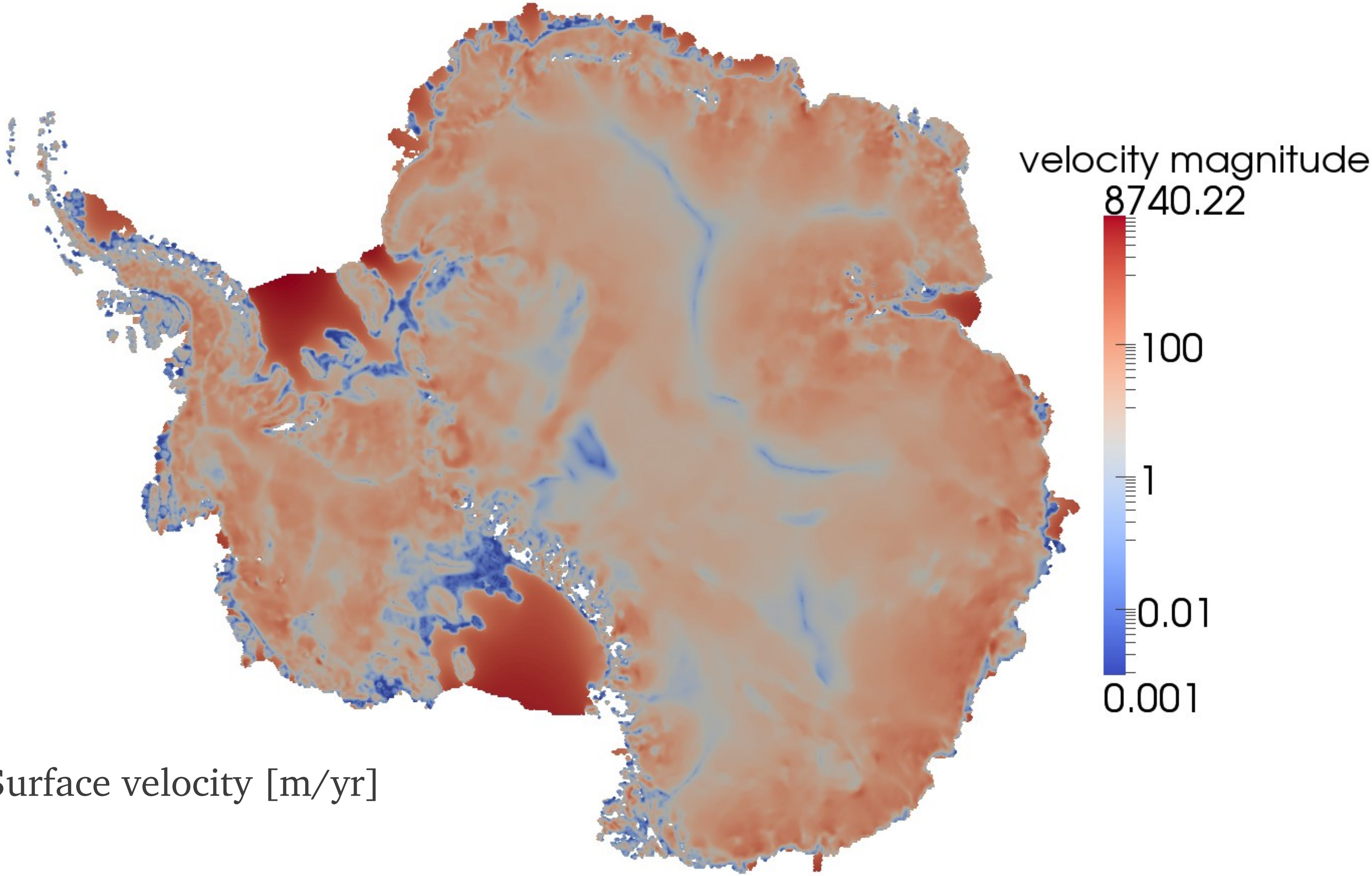


The parameter δ is decreased by LOCA from 1e-4 to 1e-9

I'm feeling lucky approach: for subsequent time steps, try solving Newton first, with a limited maximum number of iterations (say 10), if Newton does not converge, then use LOCA.

Non linear solvers
Increased Robustness with LOCA continuation method

Antarctica continuation on sliding coefficient



Surface velocity [m/yr]

Interface MPAS – FELIX

M. Hoffman (LANL), M. P.

MPAS

Land ice component

- 2D CVT mesh
(Stereographic projection)

- thickness/elevation/layers

- temperature/ice flow factor

- bedrock sliding coefficient

- solver options:
  * model (FO, L1L2, SSA, SIA)
  * nonlinear solver (Newton, Picard, JFNK)
  * Boundary condition (free-slip, no-slip, robin, coulomb)

velocity

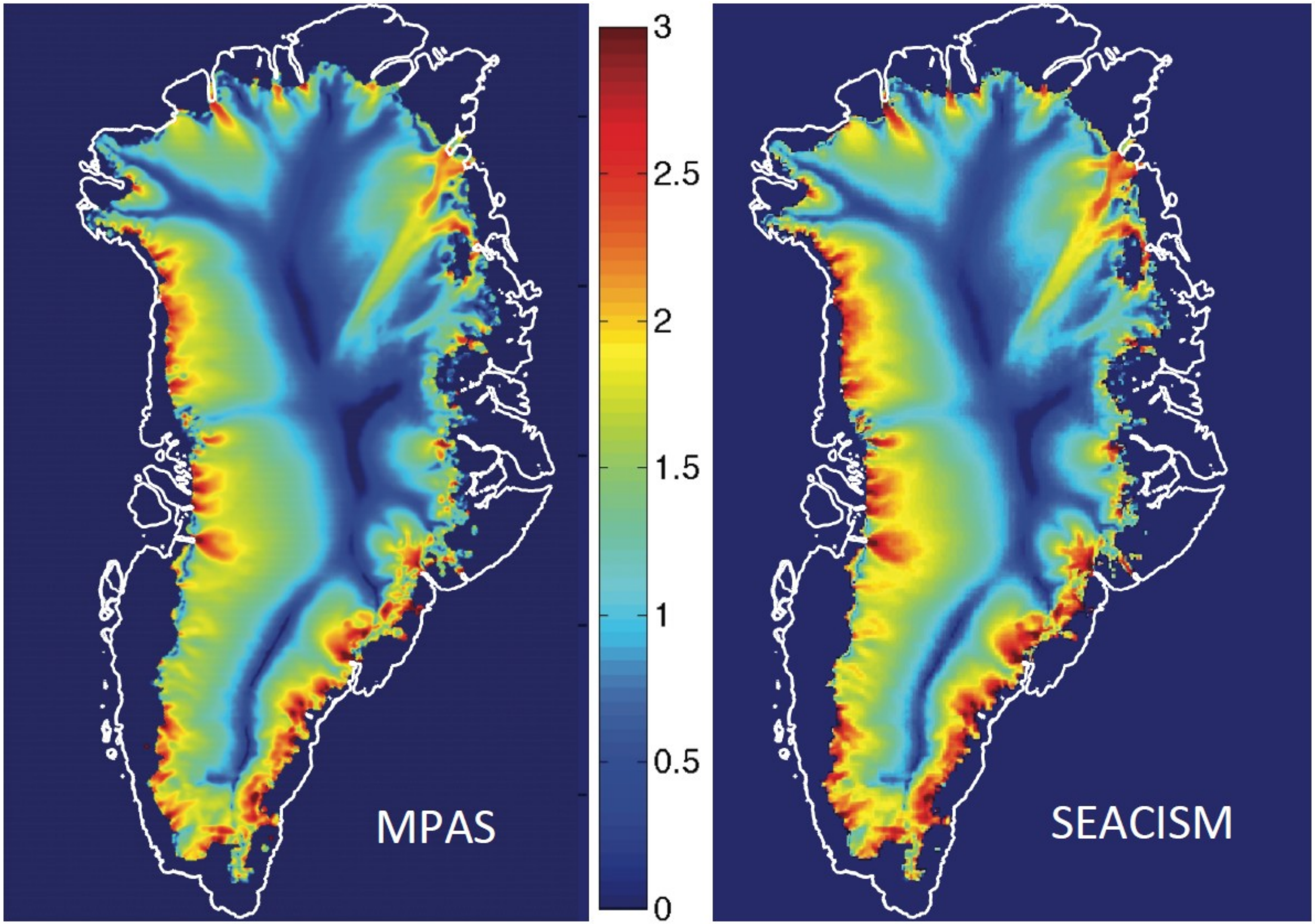
heat dissipation

viscosity

FELIX

ice-sheets component

Greenland surface velocity
Comparison CISM, MPAS-FELIX

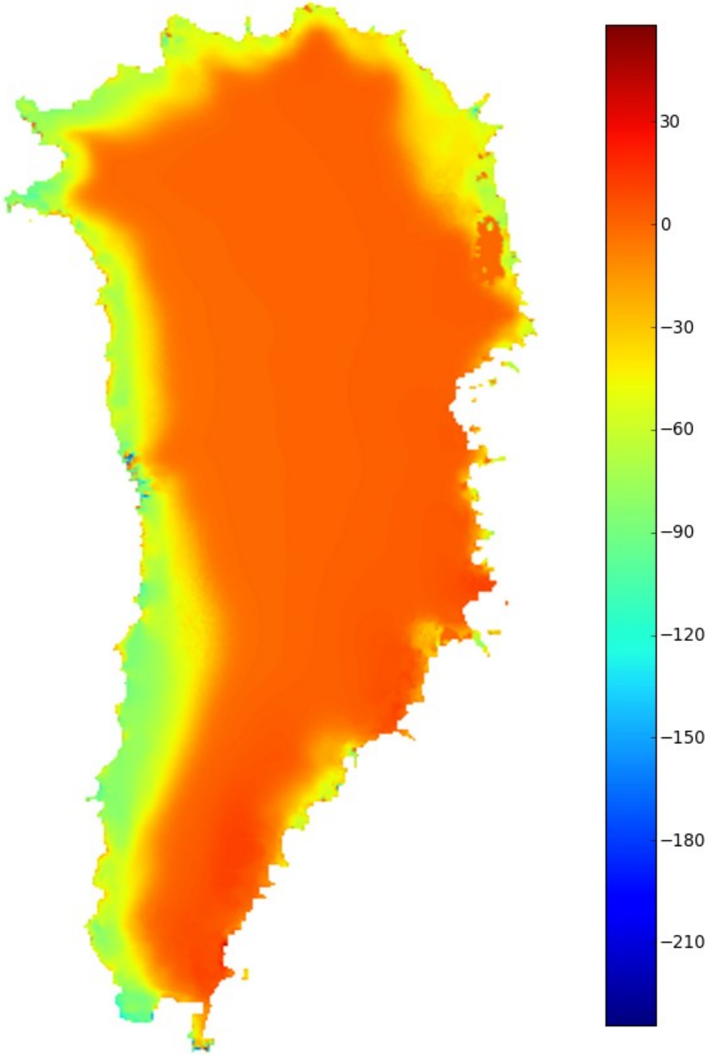


MPAS-FELIX simulations:
M. Perego (FSU) and M. Hoffman (LANL)

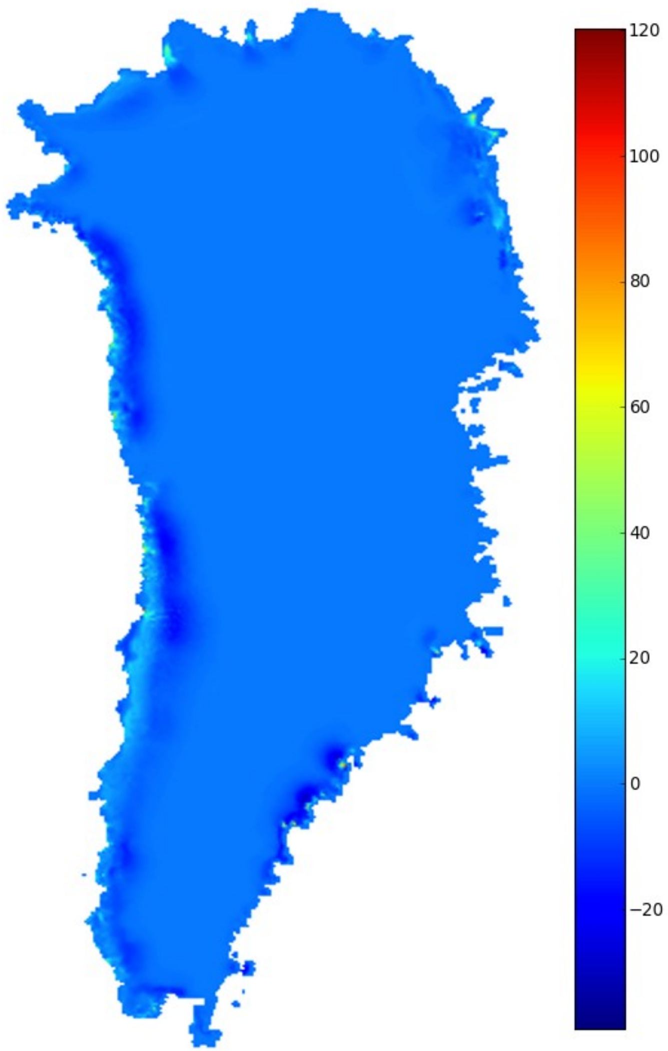
SEACISM (ORNL, SNL) simulations:
S. Price (LANL)

Ice2Sea* experiment: thickness change

**A.J. Payne et al, PNAS, submitted, to be considered for the IPCC 2014 report .*



Thickness [m] change after 100 yr



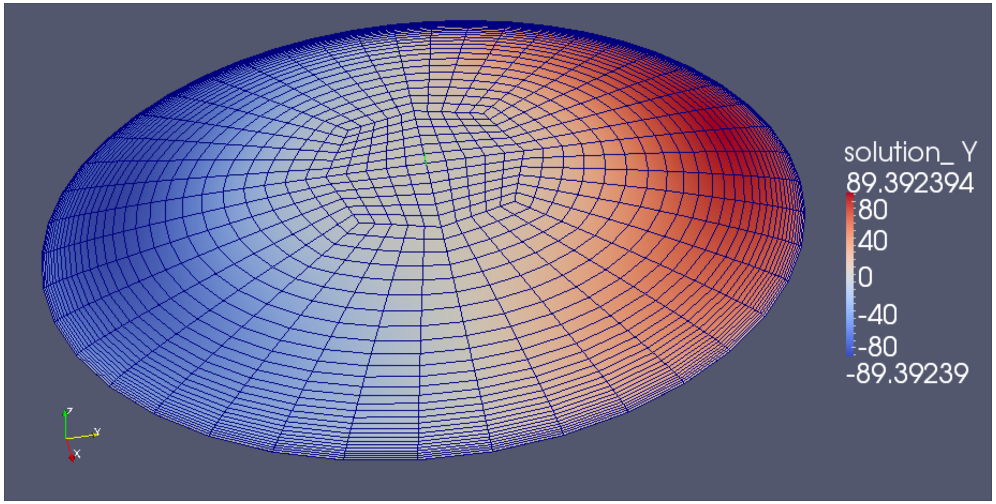
Thickness difference [m], when doubling the basal friction coefficient

UQ Problem

M. Eldred, I. Kalashnikova, A. Salinger

synthetic simulation settings

Dome problem:



Parabolic shaped dome geometry

Isothermal

Sliding b.c. at the bedrock, with friction coefficient:

$$\beta = \beta_0 + \beta_1 x + \beta_2 y + \beta_3 r, \quad r = \sqrt{x^2 + y^2}$$

Forward run: $\beta_0 = 2.9,\ \beta_1 = 0.012,\ \beta_2 = -0.002,\ \beta_3 = -0.005.$

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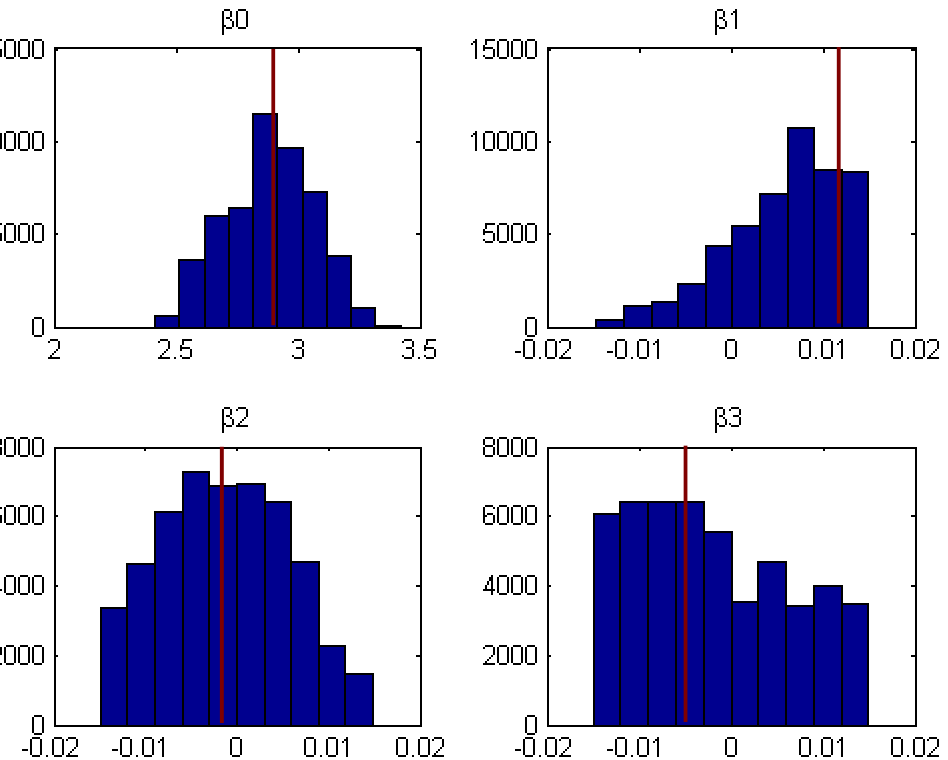
UQ Problem

M. Eldred, I. Kalashnikova, A. Salinger

Bayesian inversion Study on Dome problem

A priori distribution:
$\beta_0 \sim U(2.4, 4)$
$\beta_1,\ \beta_2,\ \beta_3 \sim U(-0.015, 0.015).$

A posteriori distribution:



	β_0	β_1	β_2	β_3
μ	2.93	0.0067	-0.0015	-0.0037
σ	0.175	0.0061	0.0062	0.0075

Bayesian inversion details:

- 100 synthetic (calibration) data points

- A polynomial chaos "reduced order model" was constructed with 200 runs

- a posteriori distribution computed using the reduced order model

Inverse Problem
Estimation of ice-sheet initial state

M. P., S. Price (LANL) and G. Stadler (UT)

The goal is to find initial conditions such that the ice is almost at thermo-mechanical equilibrium* given the geometry and the SMB.

Arthern, Gudmundsson, J. Glaciology. 2010

**Price, Payne, Howat and Smith*, PNAS 2011

Morlighem Thesis 2011

Brinkerhoff, Meierbachtol, Johnson, Harper, Annals of Glaciology, 2011

Habermann, Maxwell, Truffer, J. Glaciology. 2012

Pollard DeConto, TCD 2012

Petra, Zhu, Stadler, Hughes, Ghattas, J. Glaciology , 2012.

Inverse Problem
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Optimization Problem:
find β that minimizes the functional $\mathcal{J}$ (blue term not used in simulations)

$$\mathcal{J}_1(\beta) = \frac{1}{2}\int_{\Gamma} |\mathrm{div}(\boldsymbol{U} H) - \tau_s|^2 \, ds + \frac{1}{2}\int_{\Gamma_{top}} |\mathbf{u} - \mathbf{u}^{obs}|^2 \, ds + \mathcal{R}(\beta).$$

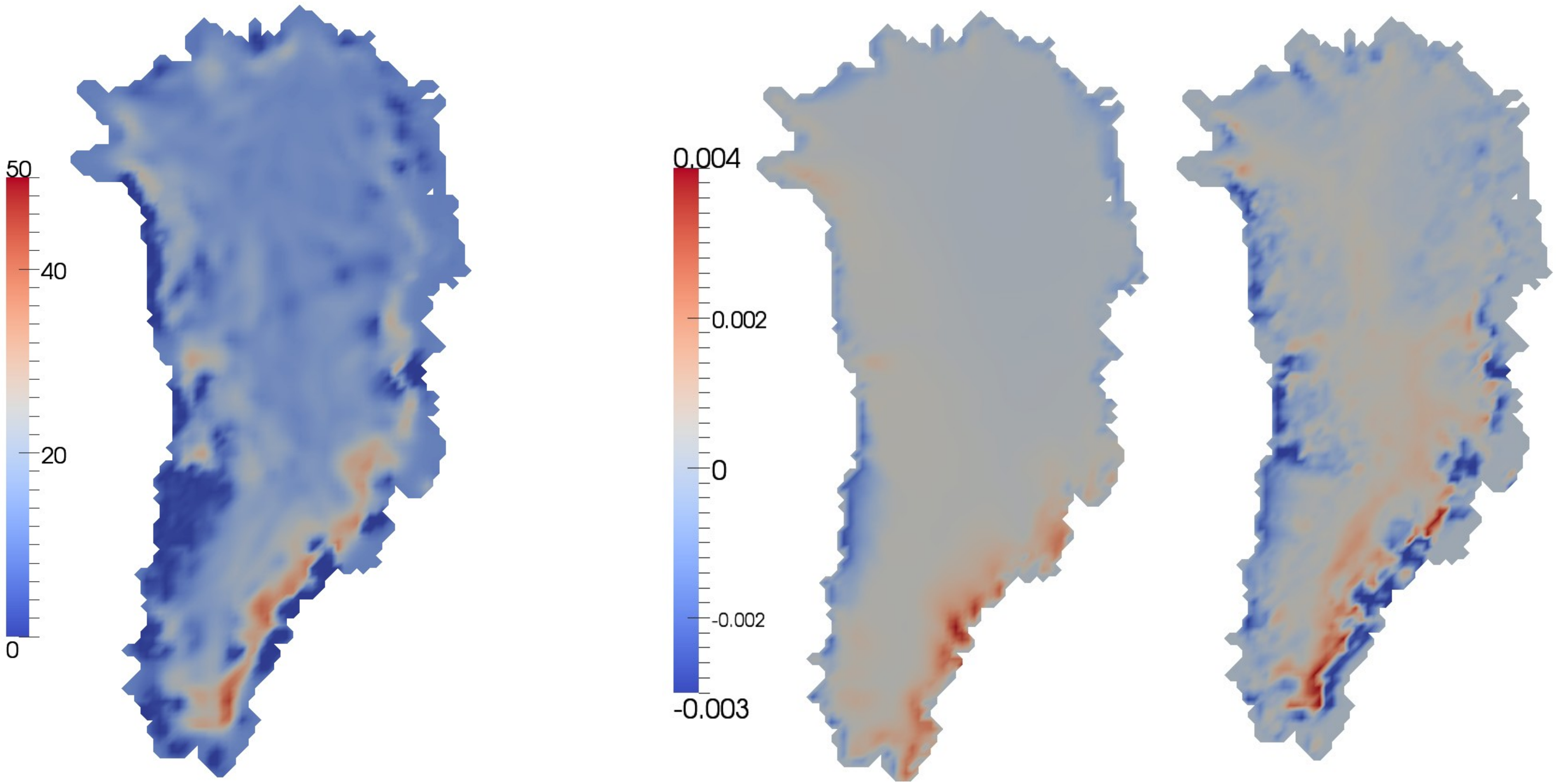
such that the ice sheet model equations (FO or Stokes) are satisfied

$\boldsymbol{U}$: computed depth averaged velocity
H: ice thickness
β: basal sliding friction coefficient
τ_s: SMB
$\mathcal{R}(\beta)$ regularization term

**Price, Payne, Howat and Smith*, PNAS 2011

Inverse Problem
Estimation of ice-sheet initial state

Preliminary Simulation, using the SMB from *Price et al,* PNAS 2011



Left: Estimated β [kPa yr m^{-1}]. Center: target SMB [km/yr].
Right: flux divergence [km/yr] computed with the estimated β.

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Inverse Problem
Estimation of ice-sheet initial state

Possible causes of the mismatch between the target and the computed SMB:

- Incorrect conditions at the lateral boundary (here we prescribe stress free b.c.).

- Incorrect temperature field (interpolated linearly between the surface temperature and the temperature at basal assumed equal to zero.

- Errors in the model (e.g. uncertainty on the Glen exponent).

- Incorrect (noisy) thickness/bedrock topography.

Possible fix for the noisy bedrock topography:*

- Modify the cost functional so that the ice thickness can slightly differ from the observed thickness:

$$\mathcal{J}_2(\beta, H) = \mathcal{J}_1(\beta) + \frac{\alpha_H}{2} \int_\Gamma |H - H^{obs}|^2 \, ds.$$

In this way we hope to filter the noise and to get thickness/bedrock topography that are consistent with the ice-sheet equations.

Morlighem, Rignot, Seroussi, Larour, Ben Dhia, Aubry, Geophysical Research Letters, 2011

Jessie Johnson, LIWG 2013

Thank you for your attention!