A comparison of two Stokes ice sheet models applied to the Marine Ice Sheet Model Intercomparison Project for plan view models (MISMIP3d)

Stephen Price[1]

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Julien Brondex[5,6], Gaël Durand[5,6], Olivier Gagliardini[5,6]

(1) Los Alamos National Laboratory
(2) Chinese Academy of Meteorological Sciences
(3) University of S. Carolina
(4) Chinese Academy of Sciences
(5) CNRS / Université Grenoble Alpes, LGGE

Supported by DOE Office of Science ASCR & BER Programs

Computing Resources:
FELIX-S: National Energy Research Scientific Computing Center (NERSC)
Elmer/Ice: Centre Informatique National de l'Enseignement Supérieur (CINES)

U.S. DEPARTMENT OF ENERGY
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*** Work inspired by discussions on first night of 2014 MISOMIP meeting ***

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Motivation

Comparison of Stokes Model Numerics

Results & Discussion

Summary & Future Work

Motivation

- Dynamic Antarctic ice sheet instability remains the largest uncertainty in projecting future sea level rise from ice sheets
- Model intercomparisons (MIPS) play an important role in designing, testing, and gaining confidence in output from ice sheet models
- MIPS are idealized, but they are becoming increasingly complex (MISMIP → MISMIP3d → MISMIP+ → MISOMIP) and lack analytic or manufactured solutions
- As such, solutions from Stokes models are commonly applied as a metric for the highest-fidelity representation of land ice dynamics
- To lend additional confidence to this practice, we conduct a careful comparison between two Stokes ice sheet models – Elmer/Ice[1] and FELIX-S[2] – applied to MISMIP3d*

[1]Gagliardini et al. (*TC*, 6, 2013) [2]Leng et al. (*JGR*, **117**, 2012)

* Currently applying to MISMIP+

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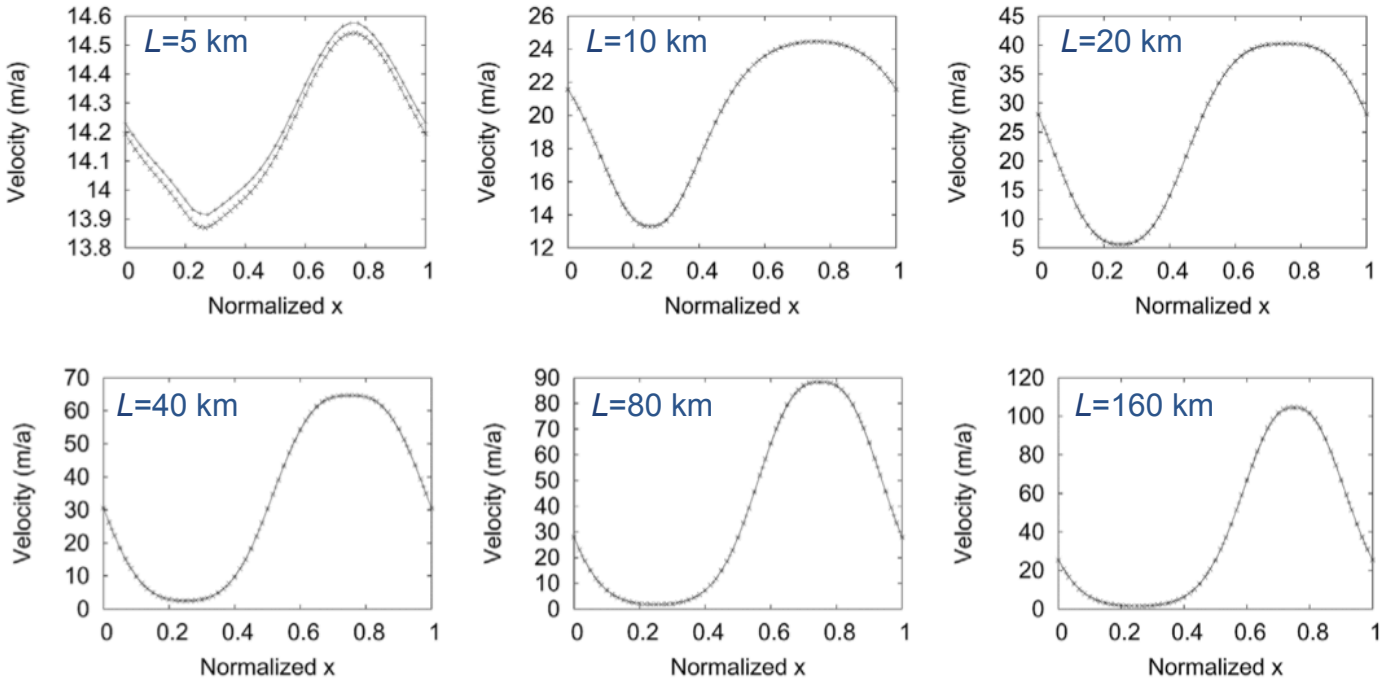
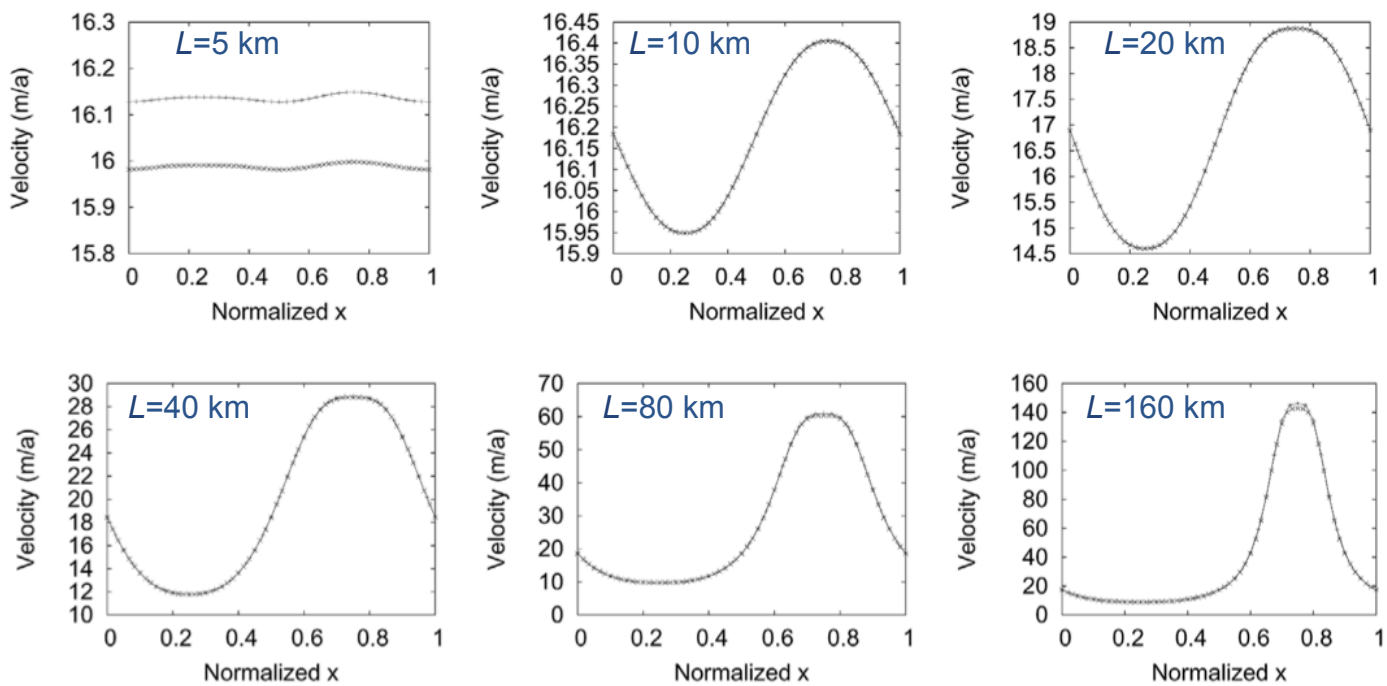
ISMIP-HOM A

FELIX-S (solid)

Elmer/Ice (dashed)



ISMIP-HOM C



Leng et al. (*JGR*, **117**, 2012)

Comparison of Stokes Numerics

feature	Elmer/Ice	FELIX-S
Element Type	hexahedra	tetrahedra (6 per hex.)
Basis Functions (u, P)	P1, P1	P2, P1 (“Taylor Hood”)
Free Surface Evolution	Gagliardini et al. (2013)	Gagliardini et al. (2013)
Nodal Contact Problem	Stokes system residual (normal *force*)	Stokes system residual (normal *stress*)
Floating / Grounded Mask	defined per node	defined per element
C at Quadrature Points	interpolated from nodal values	calculated directly
C Change Over Element **	Discontinuous (DI)	Discontinuous (DI)

** Last Grounded (LG), First Floating (FF), or Discontinuous (DI) – as discussed in Gagliardini et al. (*TC*, **10**, 2016)

FELIX-S: Stokes Solution

Given initial geometry & grounded vs. floating mask…

1. assemble FE system (including calculations of $C(x,y)$)
2. solve Stokes system (provides velocity and stress fields)
3. solve contact problem:
   1. $\sigma_{nn} > Pw$ for all 3 nodes? → mark triangle as grounded
   2. $\sigma_{nn} \leq Pw$ for for *one or more* nodes? → mark triangle as floating
4. Update mask (elements may change to/from grounded from/to floating)
5. assemble FE system (using updated calculations for $C(x,y)$)
6. solve Stokes system
7. Update geometry (possible advance/retreat due to change in mask)
8. Update grounded / floating mask:
   1. $z_b > b$ for one or more nodes? → mark triangle as floating
   2. $z_b = b$ for all 3 nodes? → mark triangle as grounded
9. Go to (3); iterate (3)-(8) until converged

FELIX-S: Stokes Solution

Given initial geometry & grounded vs. floating mask…

1. assemble FE system (including calculations of $C(x,y)$)
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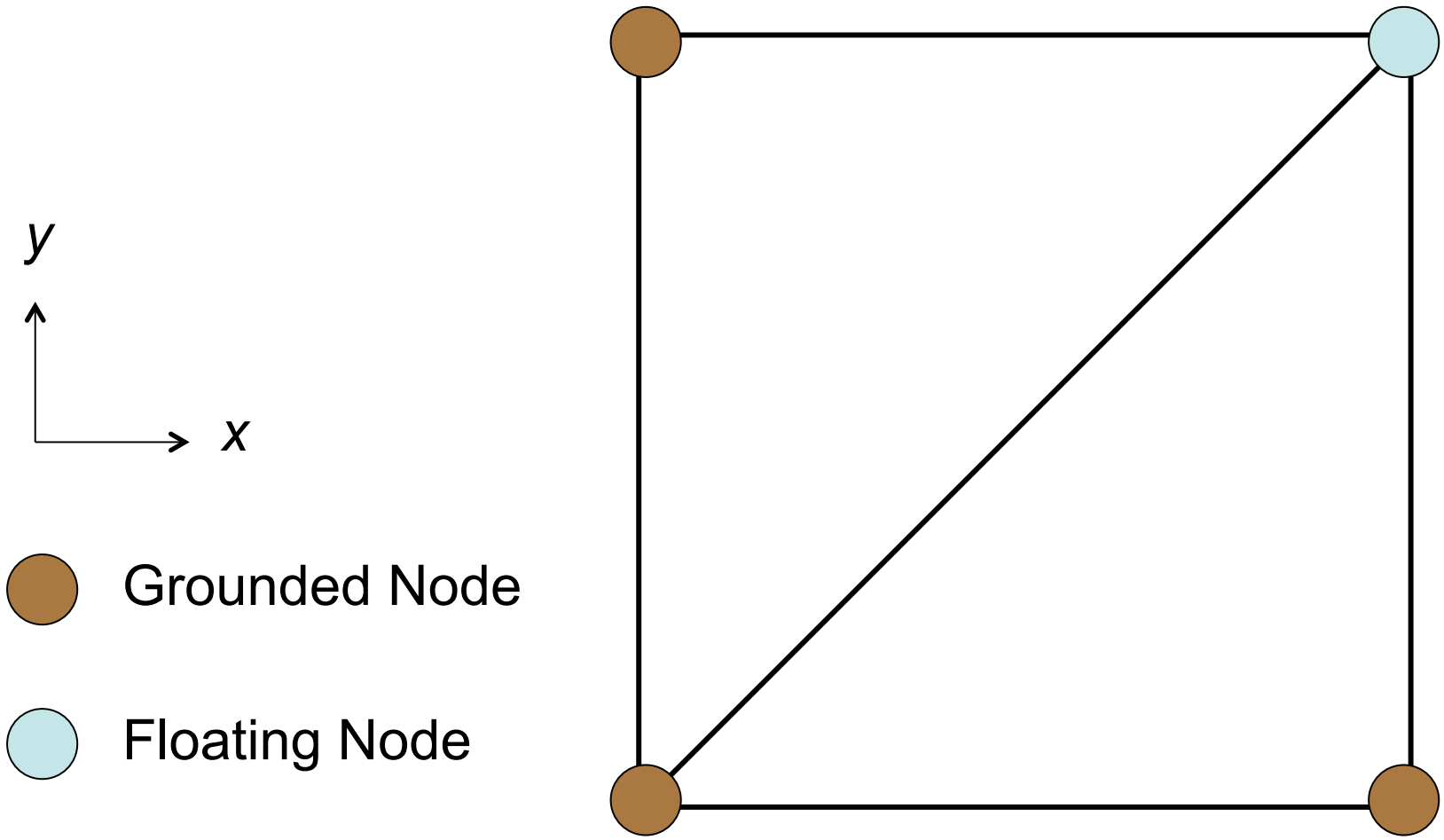
8. Update grounded / floating mask:
   1. $z_b > b$ for one or more nodes? → mark triangle as floating
   2. $z_b = b$ for all 3 nodes? → mark triangle as grounded
9. Go to (3); iterate (3)-(8) until converged

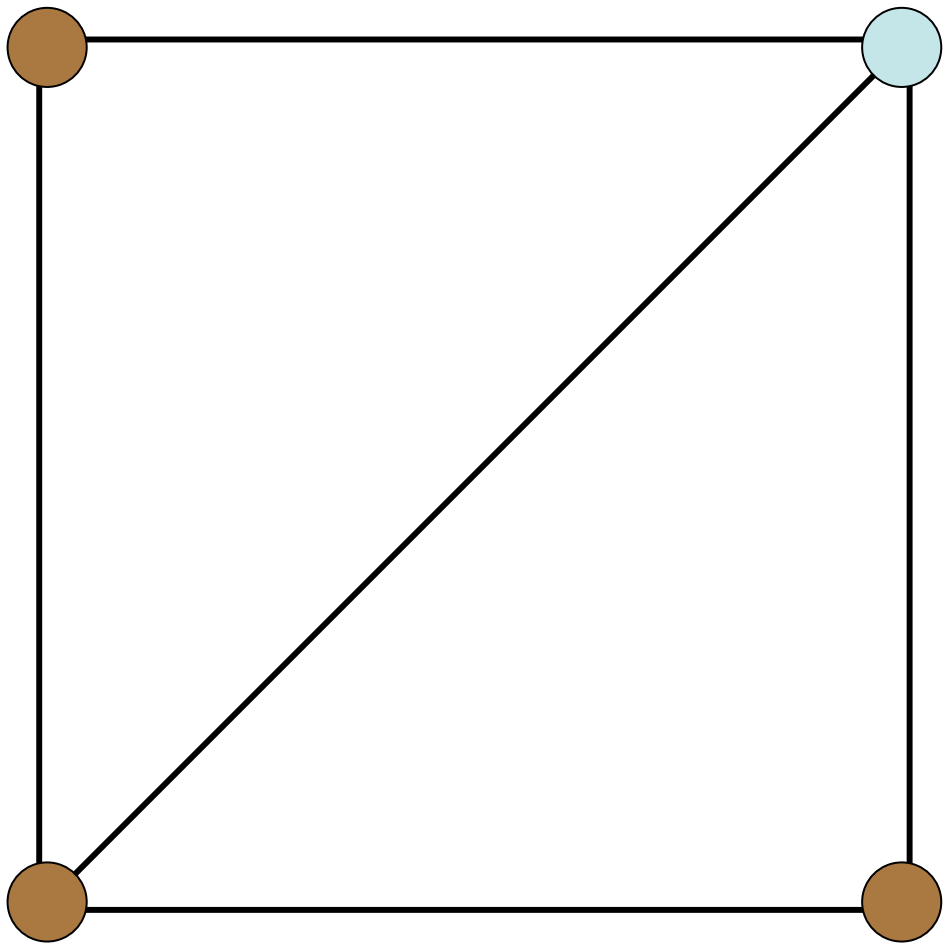
Element (FELIX-S) vs. Nodal (Elmer/Ice) Mask



Element (FELIX-S) vs. Nodal (Elmer/Ice) Mask

FELIX-S:
two floating *elements*

Elmer/Ice:
3 of 4 grounded *nodes*

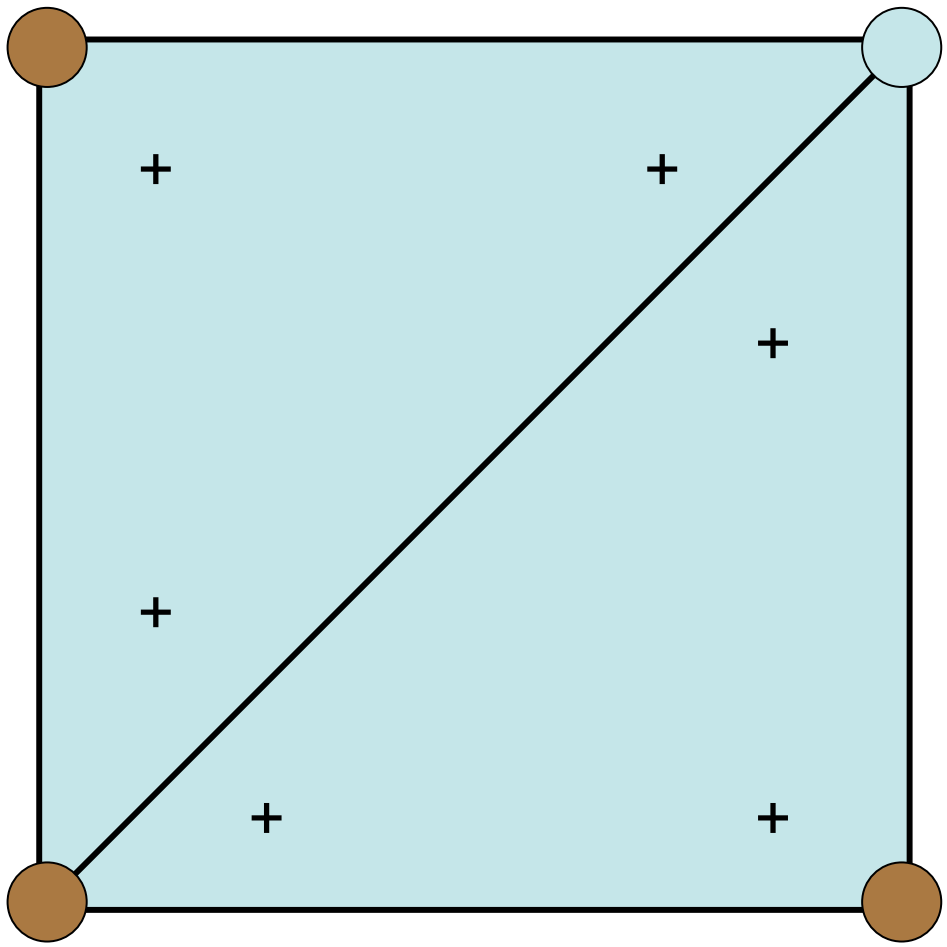


Grounded Node

Floating Node

Element (FELIX-S) vs. Nodal (Elmer/Ice) Mask

FELIX-S:
$C(x,y) = 0$ at all quadrature points



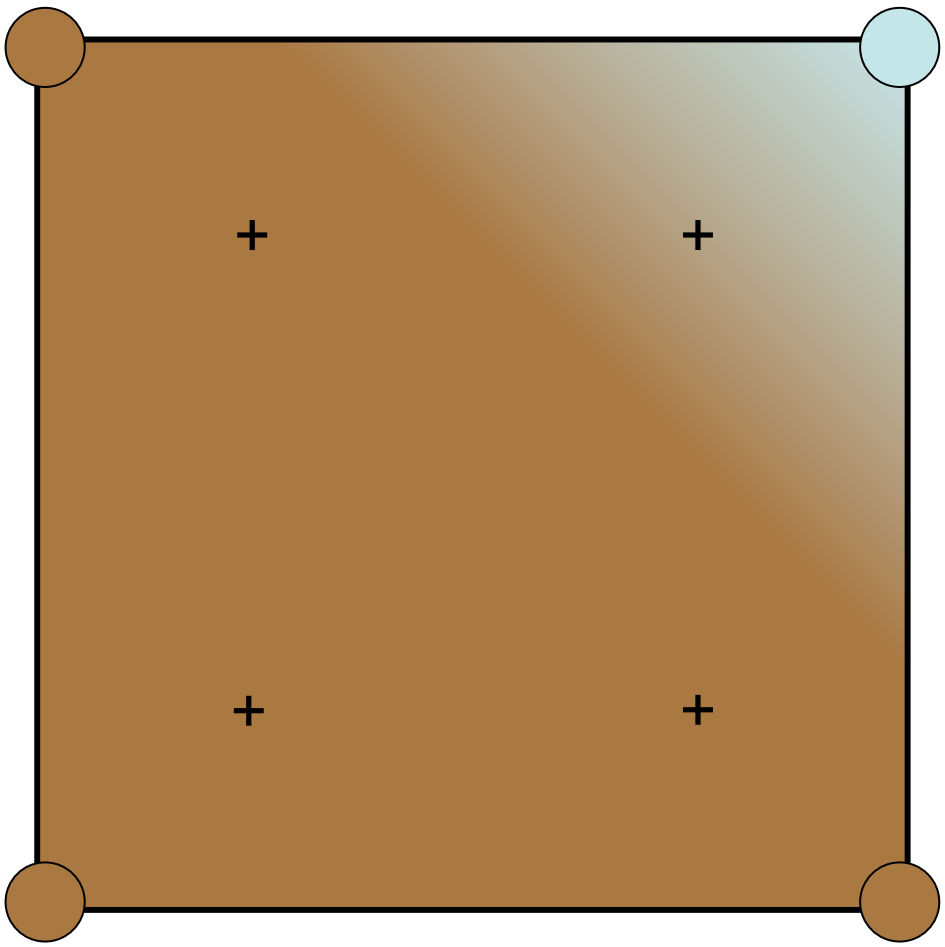
+ Quadrature Point

Grounded Node

Floating Node

Element (FELIX-S) vs. Nodal (Elmer/Ice) Mask

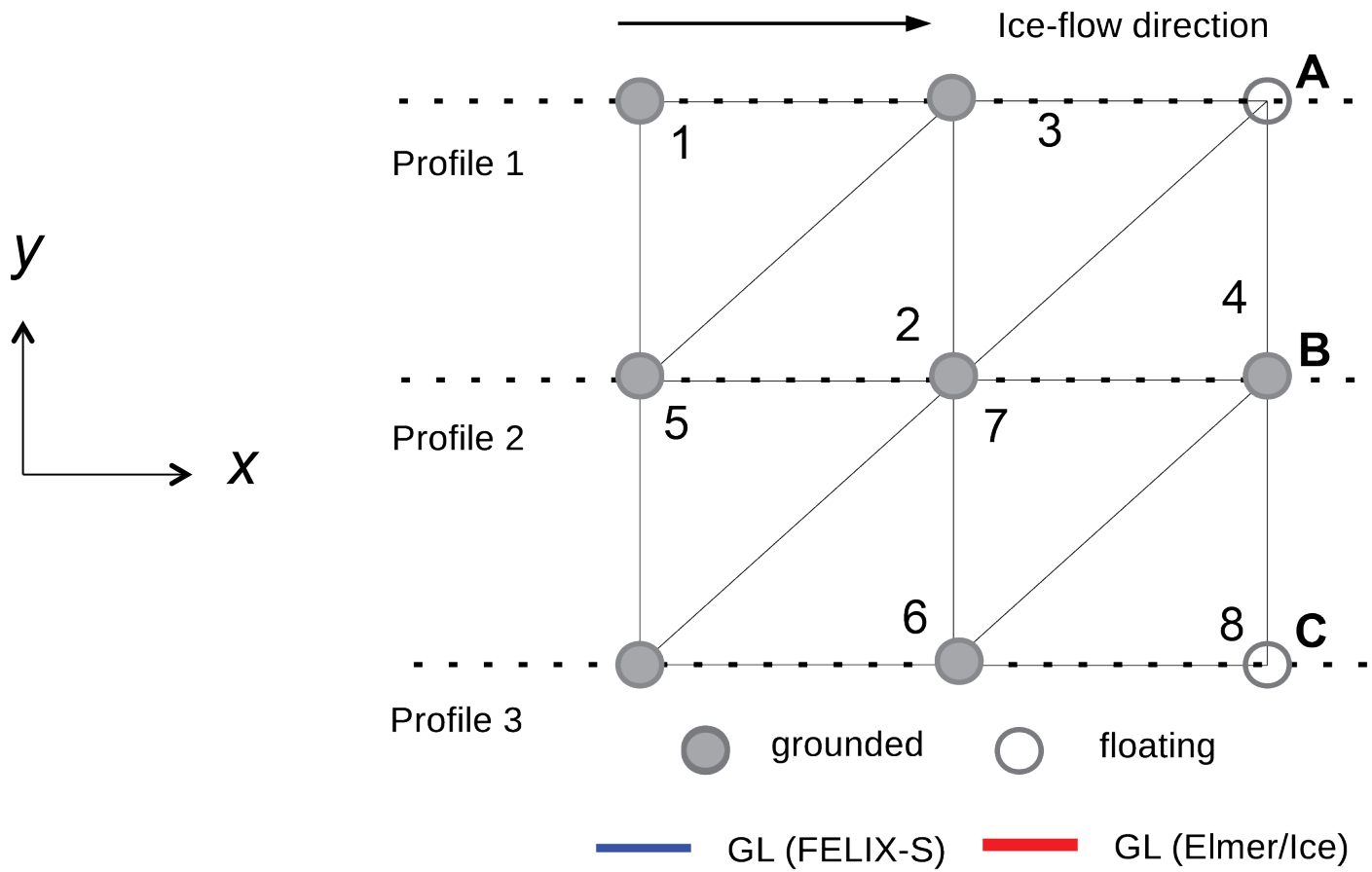
Elmer/Ice:
$C(x,y)$ = > 0 at all quadrature points

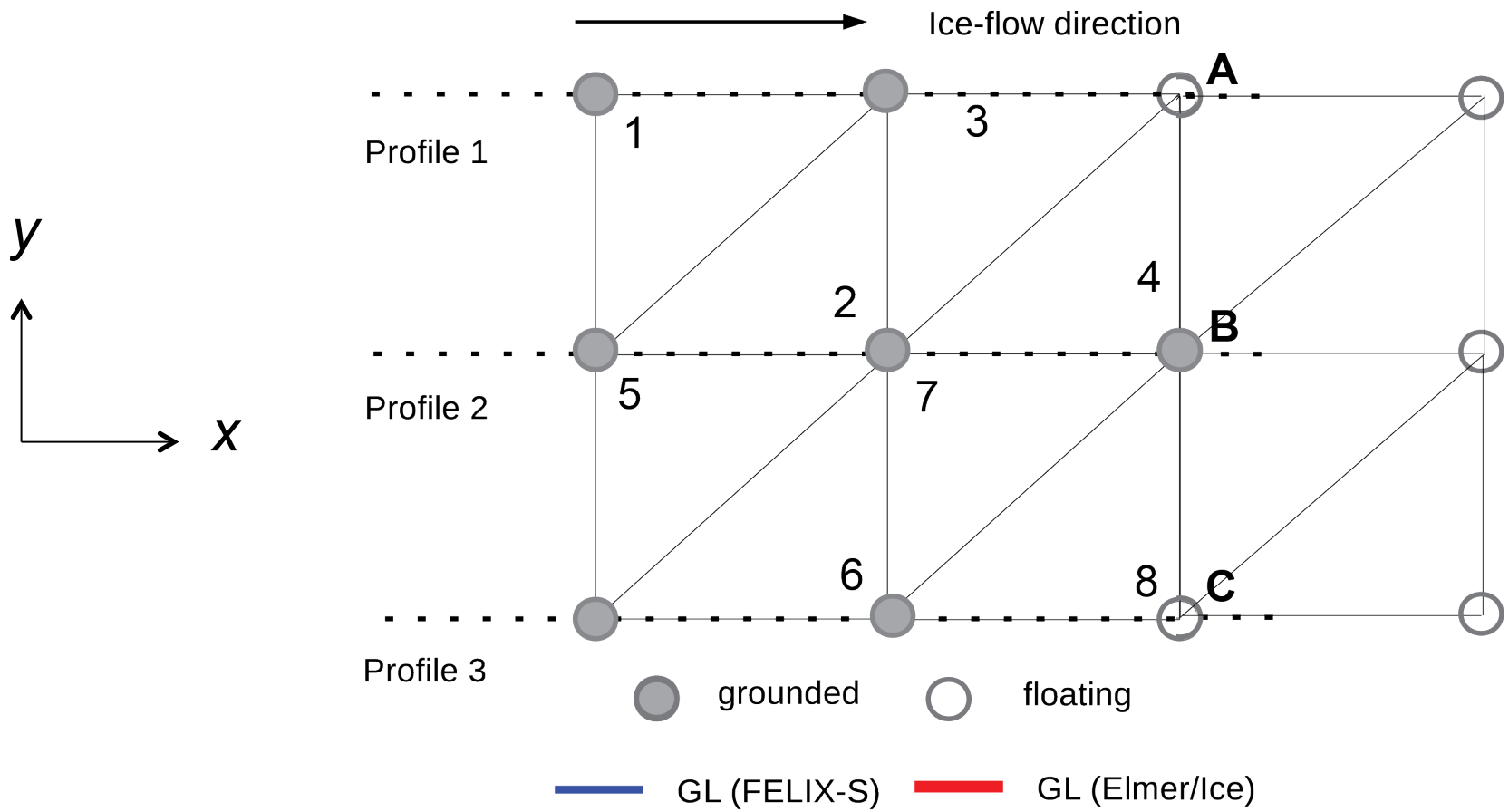


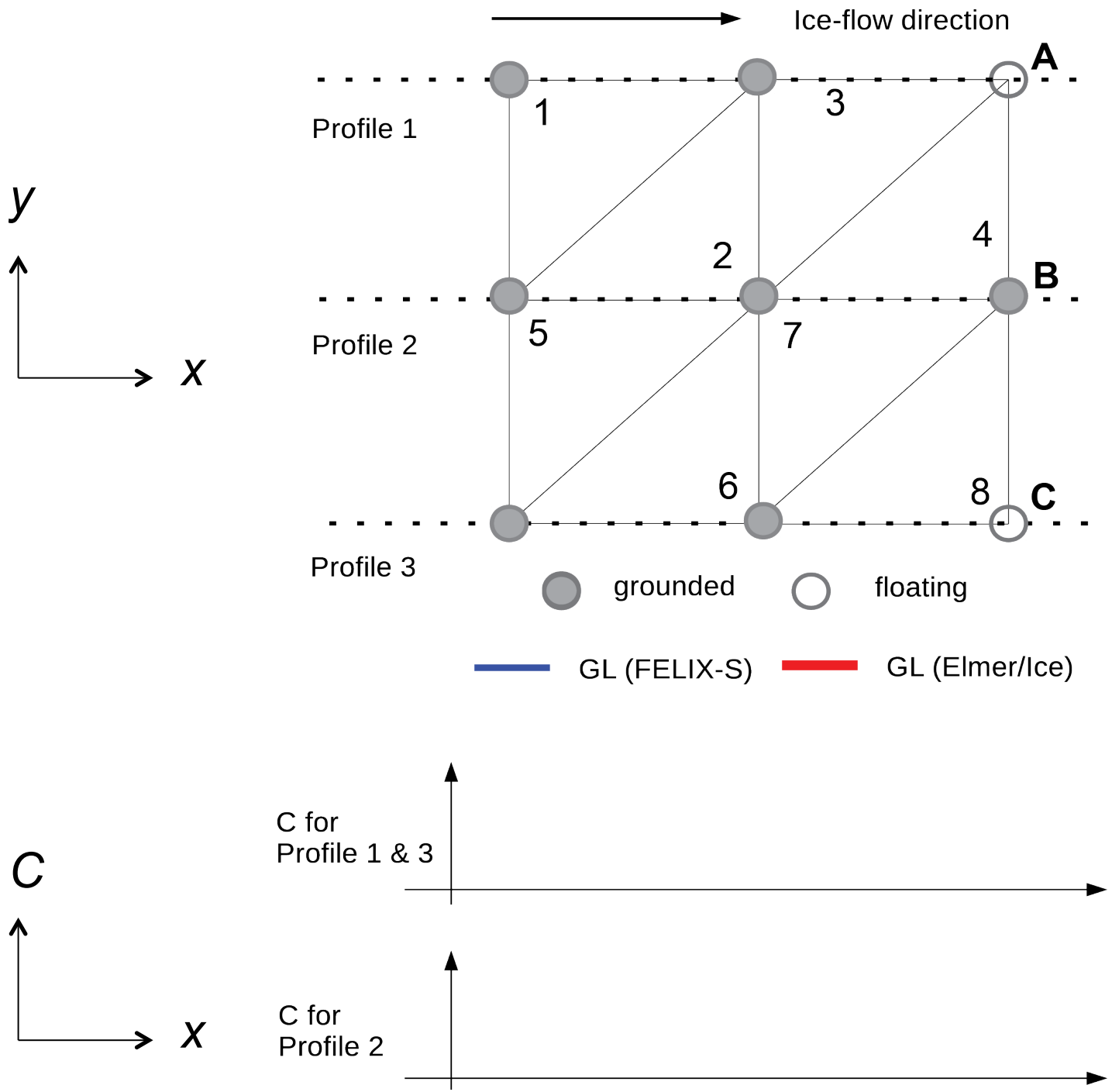
Quadrature Point

Grounded Node

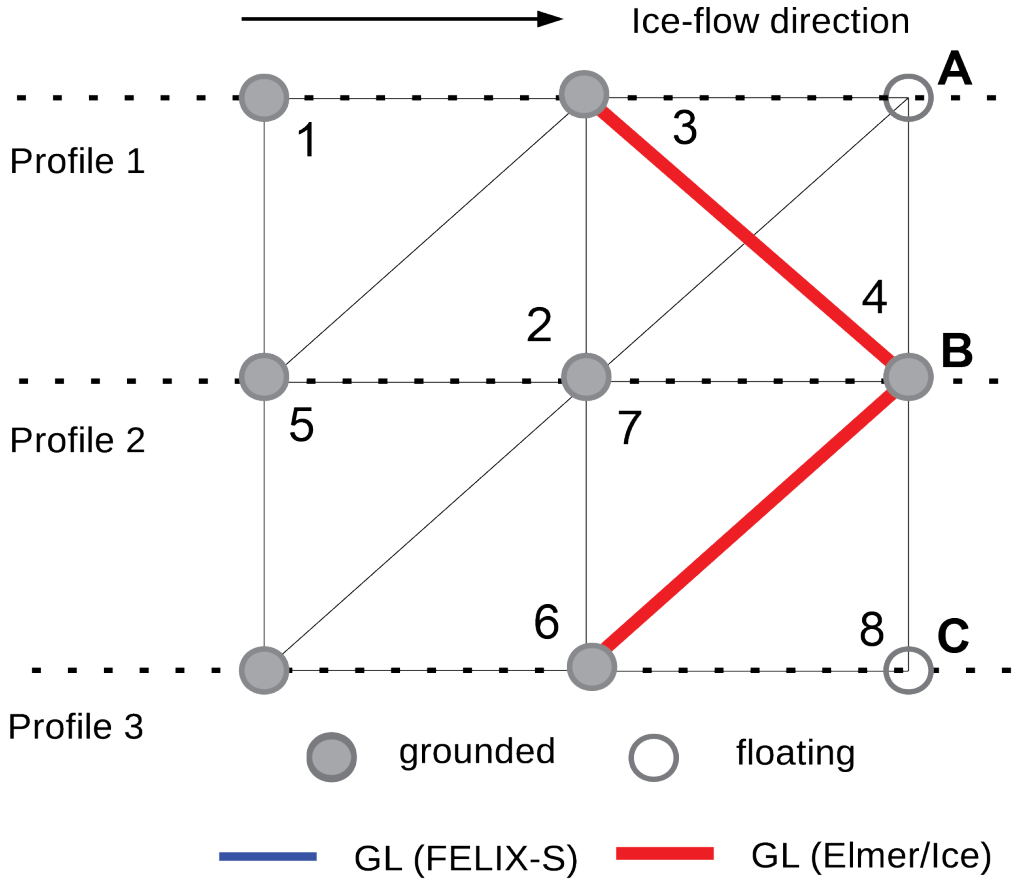
Floating Node

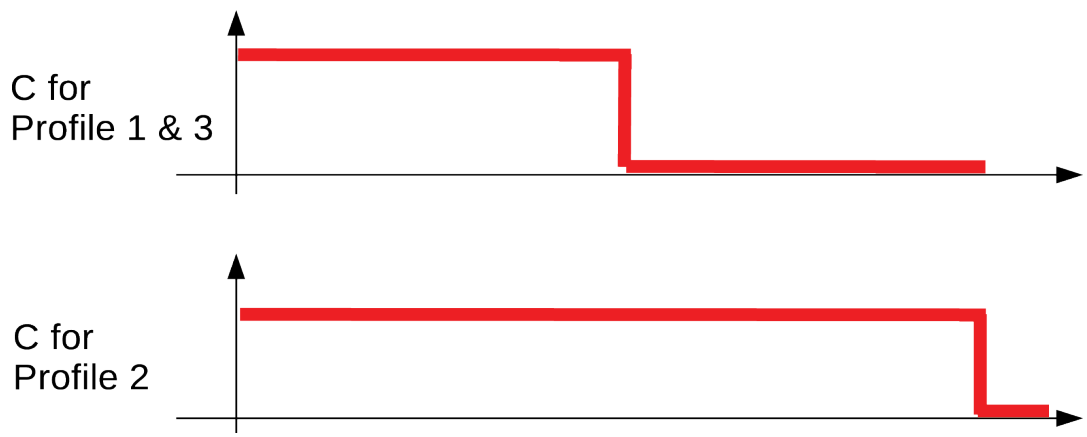

Ice-flow direction
A
B
C
1
2
3
4
5
6
7
8
Profile 1
Profile 2
Profile 3
y
x
grounded
floating
GL (FELIX-S)
GL (Elmer/Ice)


Ice-flow direction
A
B
C
1
2
3
4
5
6
7
8
Profile 1
Profile 2
Profile 3
y
x
grounded
floating
GL (FELIX-S)
GL (Elmer/Ice)

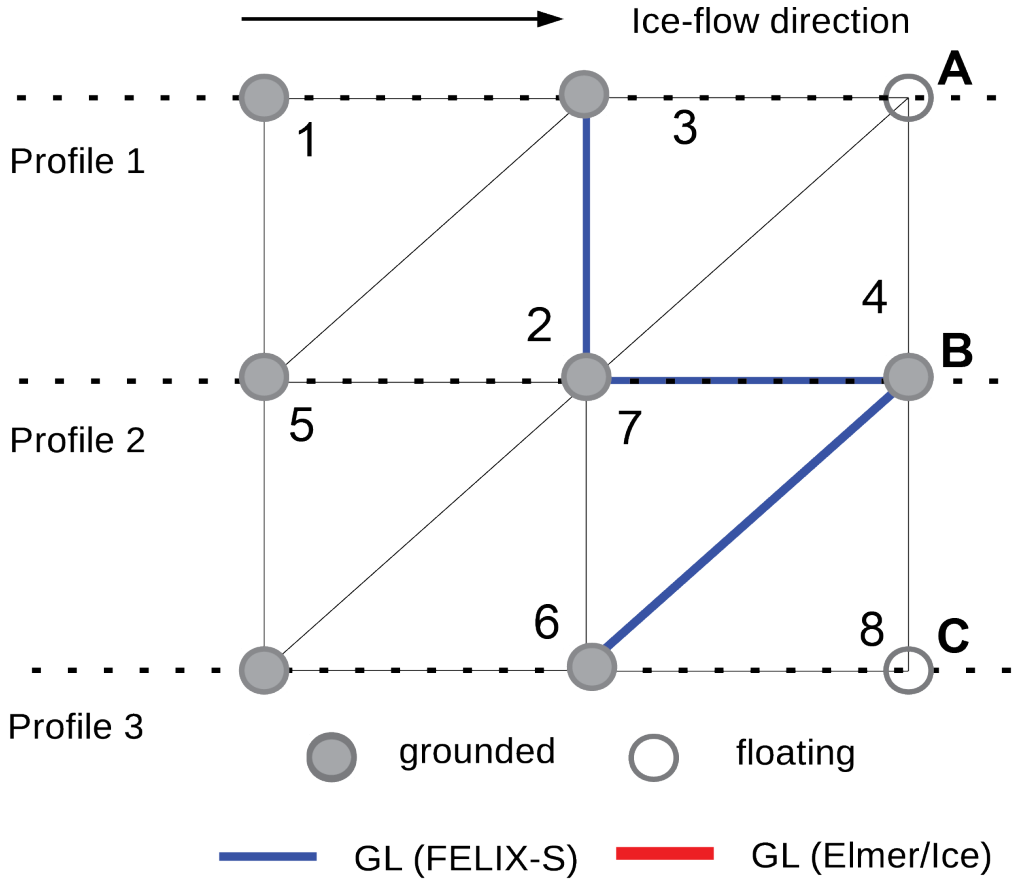

Ice-flow direction
A
B
C
1
2
3
4
5
6
7
8
Profile 1
Profile 2
Profile 3
y
x
grounded
floating
GL (FELIX-S)
GL (Elmer/Ice)
C
x
C for
Profile 1 & 3
C for
Profile 2

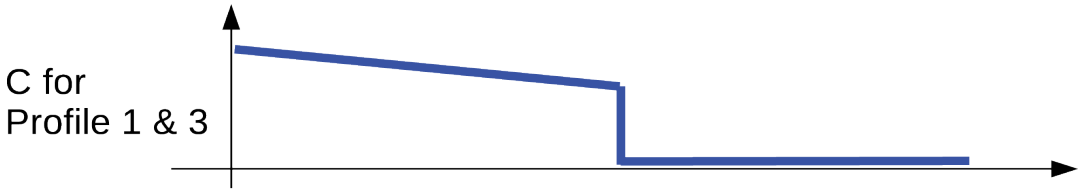
Elmer/Ice

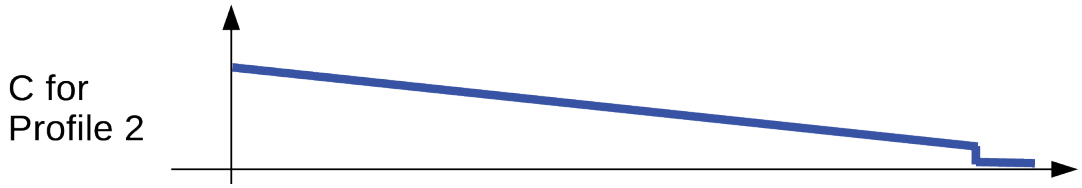

Ice-flow direction
A
B
C
1
2
3
4
5
6
7
8
Profile 1
Profile 2
Profile 3
grounded
floating
GL (FELIX-S)
GL (Elmer/Ice)

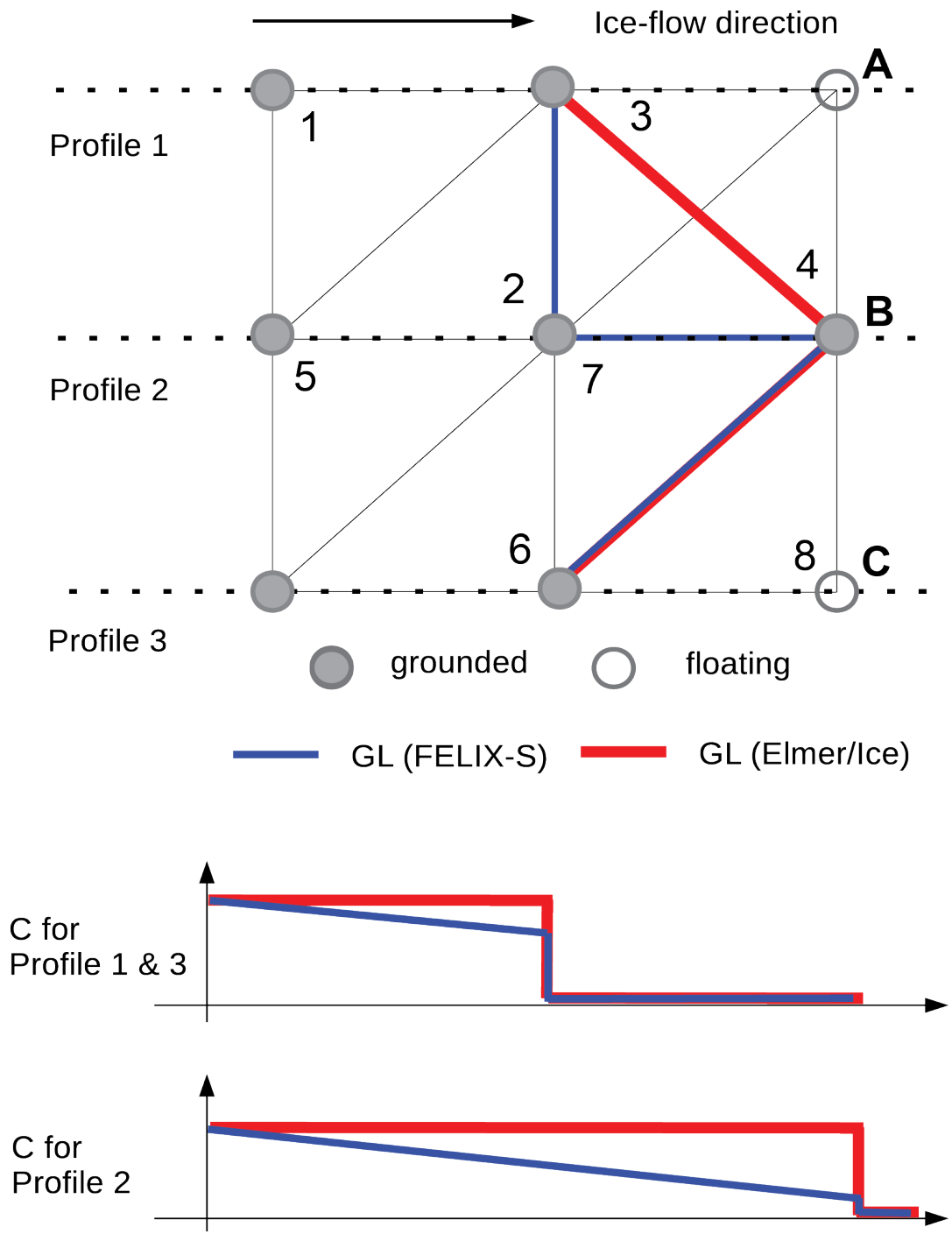

C for
Profile 1 & 3
C for
Profile 2

FELIX-S


Ice-flow direction
A
B
C
1
2
3
4
5
6
7
8
Profile 1
Profile 2
Profile 3
grounded
floating
GL (FELIX-S)
GL (Elmer/Ice)


C for
Profile 1 & 3


C for
Profile 2


Ice-flow direction
A
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5
6
7
8
Profile 1
Profile 2
Profile 3
grounded
floating
GL (FELIX-S)
GL (Elmer/Ice)
C for
Profile 1 & 3
C for
Profile 2

Elmer/Ice: Change in Friction Parameter Across GL

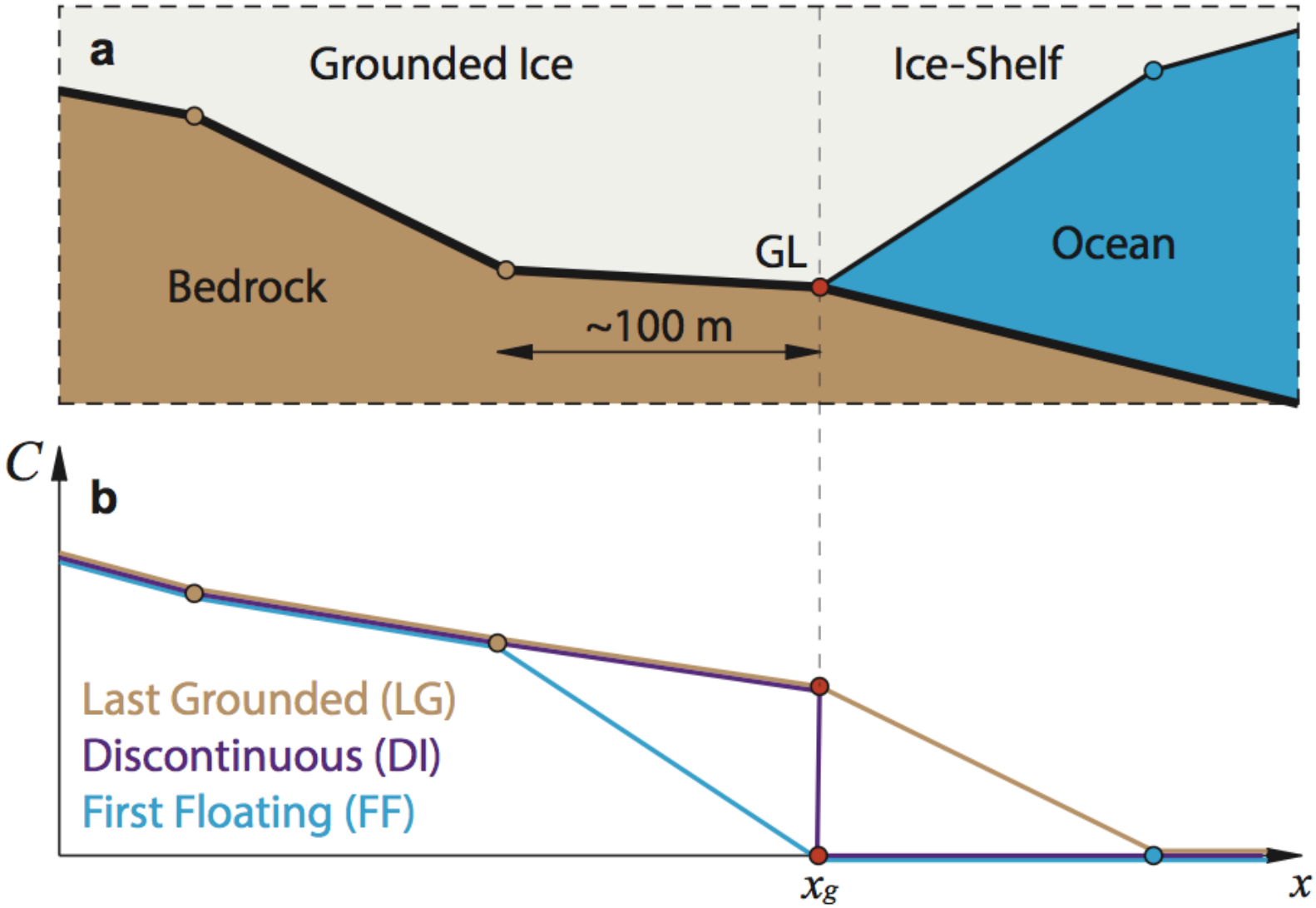
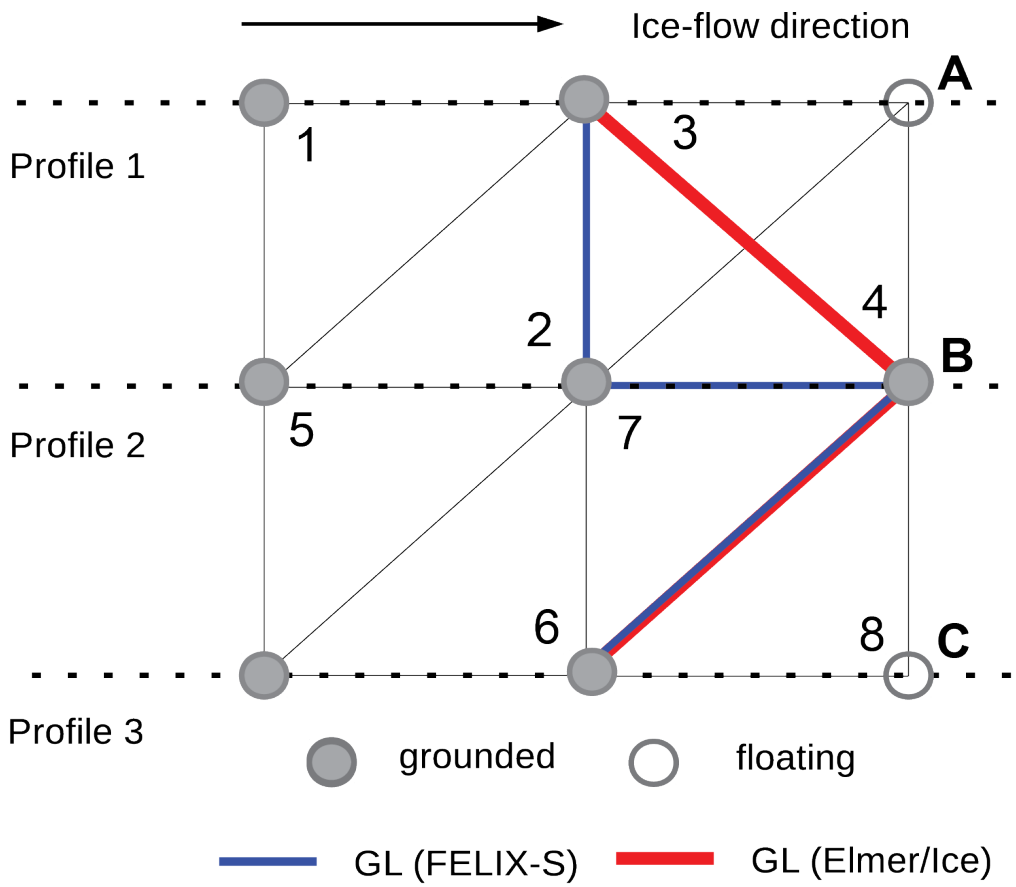


Figure from Gagliardini et al. (*TC*, **10**, 2016)


Ice-flow direction
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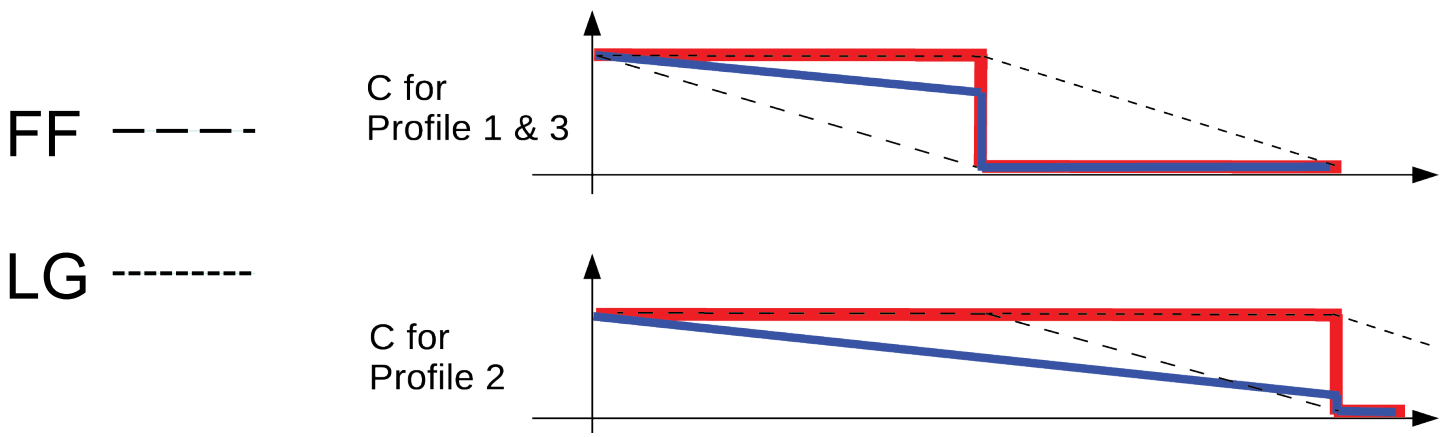
grounded
floating
GL (FELIX-S)
GL (Elmer/Ice)


FF
LG
C for
Profile 1 & 3
C for
Profile 2

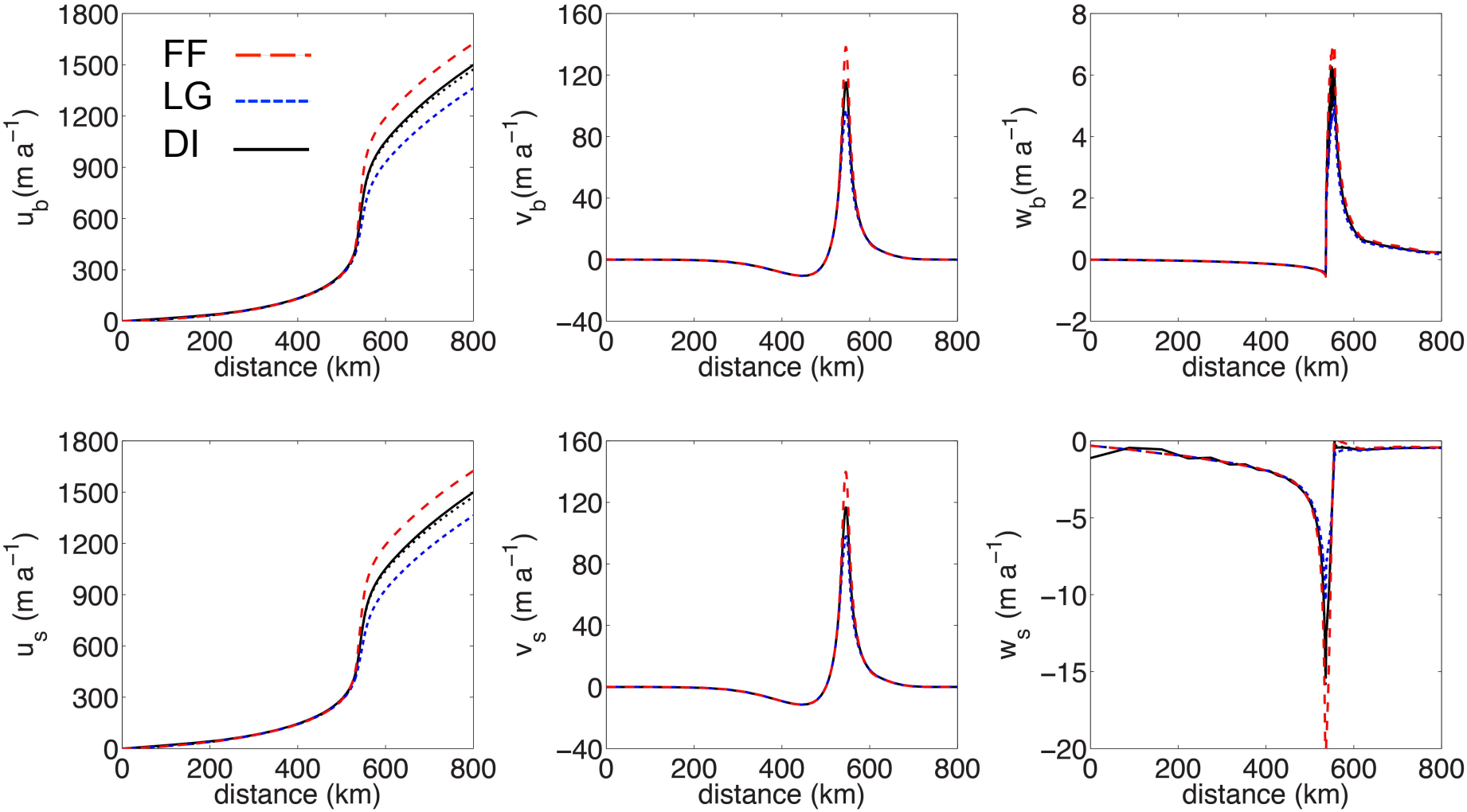
Motivation

Comparison of Stokes Model Numerics

Results & Discussion

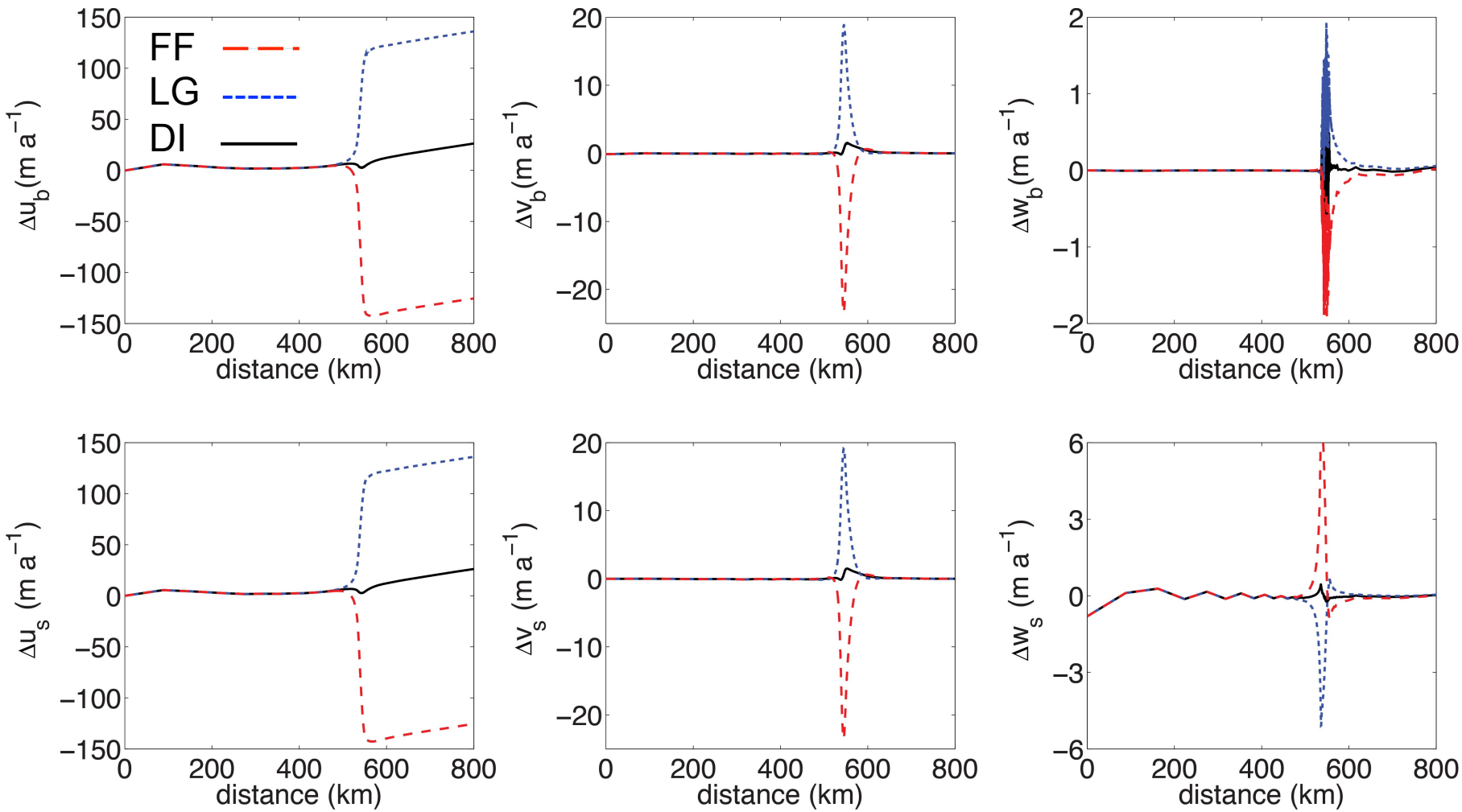
Summary & Future Work

P75D Experiment – Velocities



- Elmer/Ice nodal mask applied directly to interpolation of FELIX-S element mask to allow for as close a comparison as possible (FELIX-S prognostic masking scheme *not* applied here)

P75D Experiment – Velocity Differences



- Along-flow velocity differences are <2% (smaller at and upstream from GL)
- Across-flow and vertical velocity differences are <5%

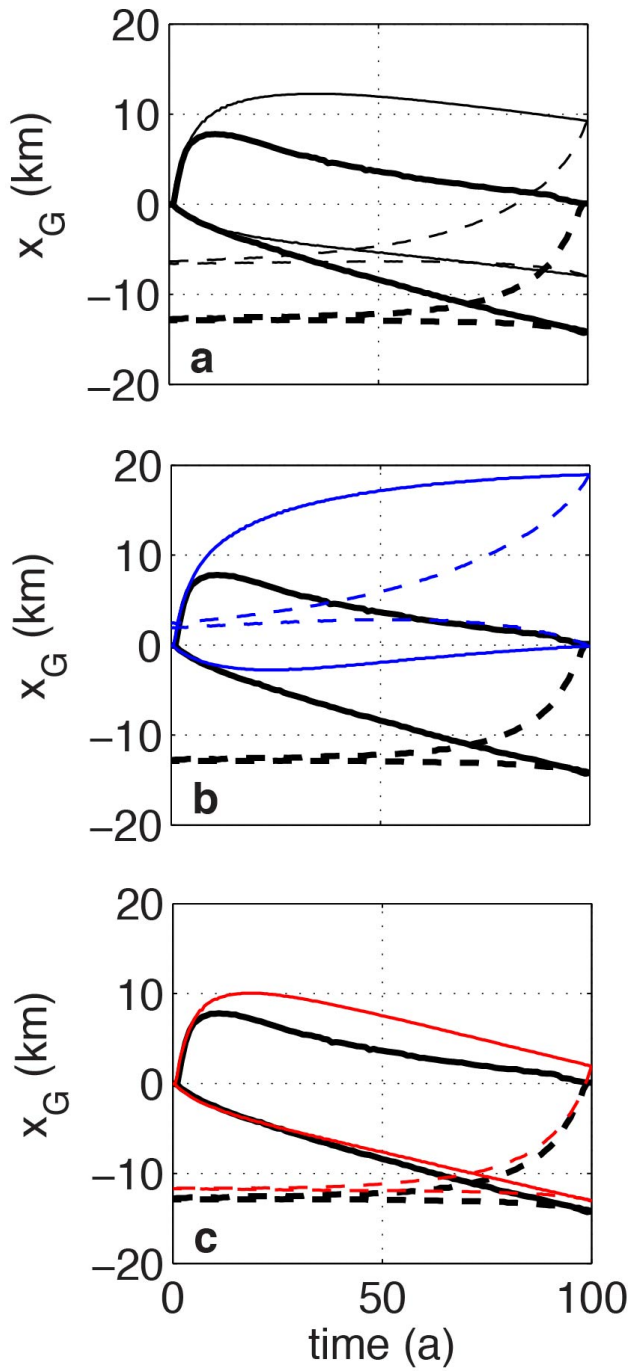
Stnd Experiment – SS GL Positions

	FELIX-S			Elmer/Ice (LG)			Elmer/Ice (DI)			Elmer/Ice (FF)		
N_y	20	40	80	20	40	80	20	40	80	20	40	80
X_{G0}	519.850	--	--	529.550	--	--	526.800	--	--	522.350	--	--
ΔX_{GC}	0.100	4.350	9.400	18.950	16.350	15.050	9.250	10.825	11.950	1.950	6.425	9.900
ΔX_{GM}	-14.050	-8.950	-6.250	-0.100	-2.750	-3.850	-8.00	-7.050	-6.250	-13.050	-10.250	-7.850

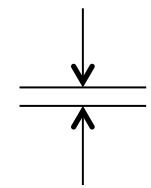
* FELIX-S & Elmer/Ice (FF) SS GL's differ by ~2 km

P75S ——

P75R - - - - -



Elmer/Ice truncation error



Elmer/Ice DI: ——

Elmer/Ice LG: ——

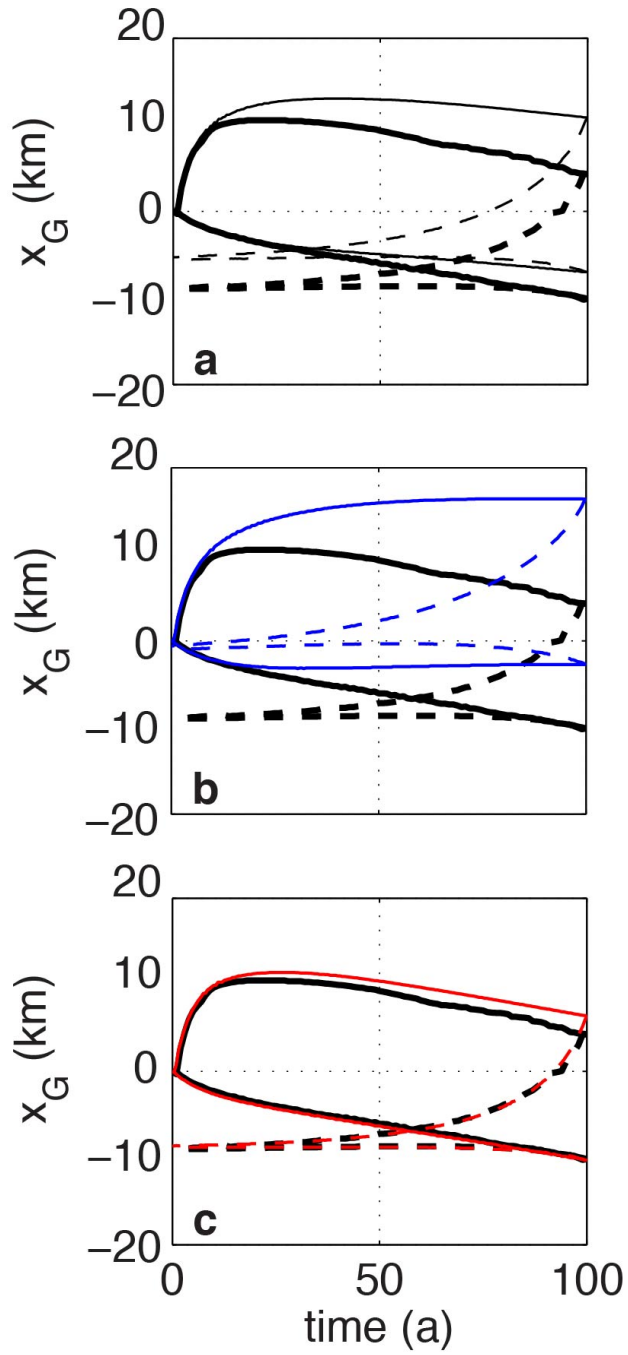
Elmer/Ice FF: ——

FELIX-S DI: ——

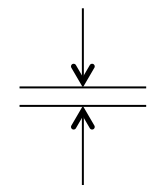
dx=50m
Ny=20
dy=2500 m

P75S ——

P75R - - - - -



Elmer/Ice truncation error



Elmer/Ice DI: ——

Elmer/Ice LG: ——

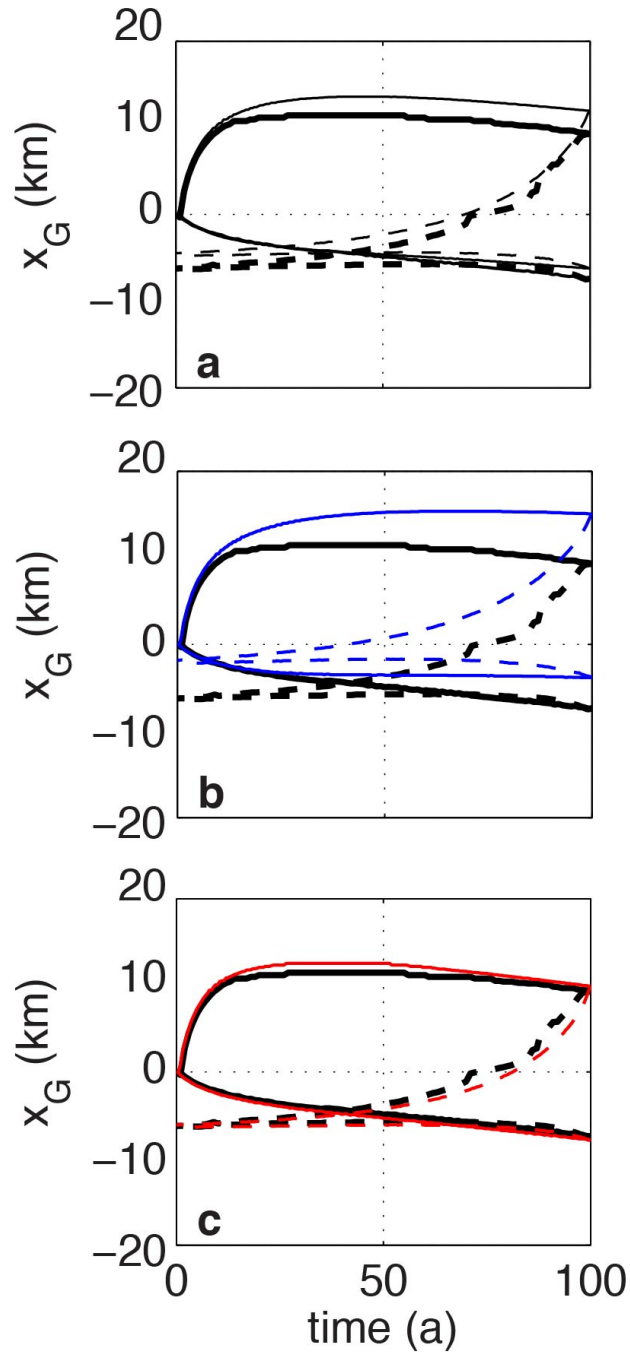
Elmer/Ice FF: ——

FELIX-S DI: ——

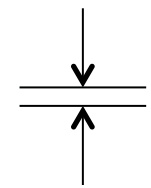
dx=50m
Ny=40
dy=1250 m

P75S ——

P75R - - - - -



Elmer/Ice truncation error



Elmer/Ice DI: ——

Elmer/Ice LG: ——

Elmer/Ice FF: ——

FELIX-S DI: ——

dx=50m
Ny=80
dy=625 m

P75S,D Experiments – Change in GL Position

	FELIX-S			Elmer/Ice (LG)			Elmer/Ice (DI)			Elmer/Ice (FF)		
N_y	20	40	80	20	40	80	20	40	80	20	40	80
X_{G0}	519.850	--	--	529.550	--	--	526.800	--	--	522.350	--	--
ΔX_{GC}	**0.100**	**4.350**	**9.400**	18.950	16.350	15.050	9.250	10.825	11.950	**1.950**	**6.425**	**9.900**
ΔX_{GM}	**-14.050**	**-8.950**	**-6.250**	-0.100	-2.750	-3.850	-8.00	-7.050	-6.250	**-13.050**	**-10.250**	**-7.850**

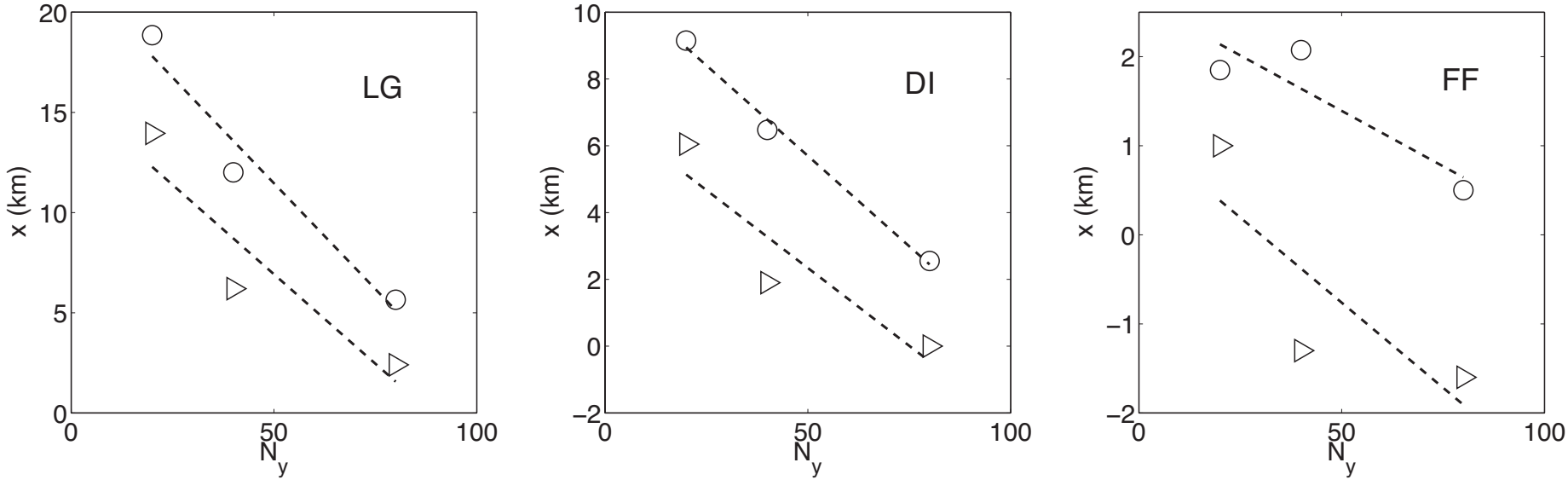
* Changes in ΔGL with increasing resolution differ by ~2 km

P75S,D Experiments – Change in GL Position

	FELIX-S			Elmer/Ice (LG)			Elmer/Ice (DI)			Elmer/Ice (FF)		
N_y	20	40	80	20	40	80	20	40	80	20	40	80
X_{G0}	519.850	--	--	529.550	--	--	526.800	--	--	522.350	--	--
ΔX_{GC}	0.100	4.350	**9.400**	18.950	16.350	15.050	9.250	10.825	**11.950**	1.950	6.425	**9.900**
ΔX_{GM}	-14.050	-8.950	**-6.250**	-0.100	-2.750	-3.850	-8.00	-7.050	**-6.250**	-13.050	-10.250	**-7.850**

* For DI & FF ΔGL's at highest resolution agree to within ~2 km

Change in ΔGL ($\Delta GL_{Elmer} - \Delta GL_{FELIX}$) with increasing grid resolution



centerline

margin

Summary & Future Work

- FELIX-S and Elmer/Ice differ in numerics and implementation:
  - element types, basis functions
  - floating vs. grounded masking scheme
  - calculation of sliding coefficient
- … but no clear reason to choose one model over the other

- While both models use DI, masking differences likely to result in FELIX-S being closer to Elmer/Ice when latter using “FF” implementation

- **P75D:** velocity differences are small (≤2-5%)
- **Stnd:** SS GL differences are ~2 km (FF)
- **P75S,R:**
  - Elmer/Ice truncation error ~2-3 km
  - ΔGL for both models agree to within ~2 km with Δ resolution (FF)
  - at highest resolution, ΔGL differences agree to within ~2 km (FF,DI)

- Currently working on MISMIP+ experiments with FELIX-S